EFFECTS OF DELAYED COMPENSATION ON SERVOMECHANISM SYSTEMS

ROBERT J. SKERRETT

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ON SERVOMECHANISM SYSTEMS

by

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Lieutenant, United States Navy

Submitted in partial fulfillment of
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ABSTRACT

The purpose of this thesis is to improve the transient response of servomechanism systems. This is accomplished by employing normal methods of compensation such as tachometer feedback and lead network systems. The response of the system is improved by introducing these compensations after the system has begun to respond to the forcing function.

This paper also considers the design problems that might be encountered in adapting this method of control to an existing servomechanism system.

ACKNOWLEDGMENT

The author wishes to express his appreciation to Doctor G. J. Thaler for the invaluable aid and counsel which he has offered during the preparation of this thesis.

TABLE OF CONTENTS

Page

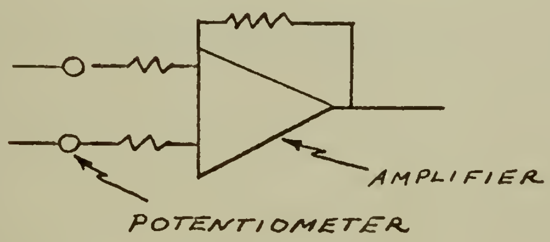
LIST OF SYMBOLS

θ_i	Input signal
θ_o	Response of system
$\dot{\theta}_o$	Output Velocity
$\mathcal{E}$	Error signal
$\mathcal{T}_d$	Torque developed
T	Time lag before introduction of damping
τ	Time constant
K_1	Forward loop amplification
h	Feedback amplification
ζ_1	Damping ratio without compensation
ζ_2	Damping ratio with compensation
A	System constant
B	Magnitude of step input
Condition I	The condition that exists prior to the introduction of the compensation
Condition II	The condition that exists after the introduction of the compensation
G_{o1}	Open loop transfer function of Condition I
G_{c1}	Closed loop transfer function of Condition I
G_{o2}	Open loop transfer function of Condition II
G_{c2}	Closed loop transfer function of Condition II
ρ_1, ρ_2	Poles of G_{o1}
r_1, r_2	Roots of G_{c1}
P_1, P_2	Poles of G_{o2}
R_1, R_2	Roots of G_{c2}

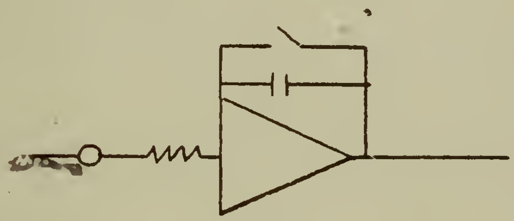
× Pole of G_{O1} OR G_{O2} (on root locus plot)

○ Root of G_{C1} OR G_{C2} (on root locus plot)

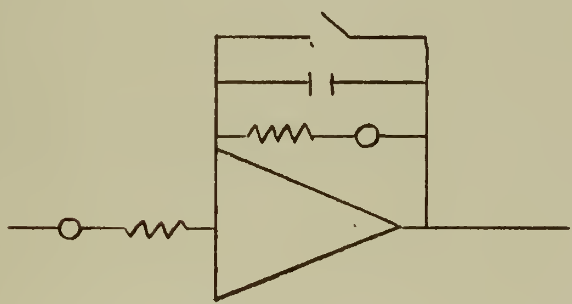
□ Zero of LEAD NETWORK (on root locus plot)



Adder



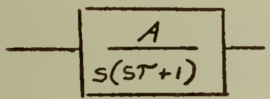
Integrator



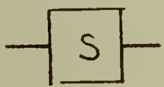
Transfer function setup

Adders

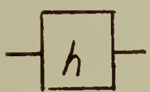
Forward loop amplifier



Motor

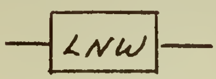


Tachometer



Feedback amplifier

Delay switch



Lead network

I GENERAL INTRODUCTION

A) PURPOSE

The purpose of this paper is to investigate various methods of improving the response of servo mechanism systems in the time domain.

B) BACKGROUND

It is the purpose of any servo systems to accurately duplicate any signal applied to it.

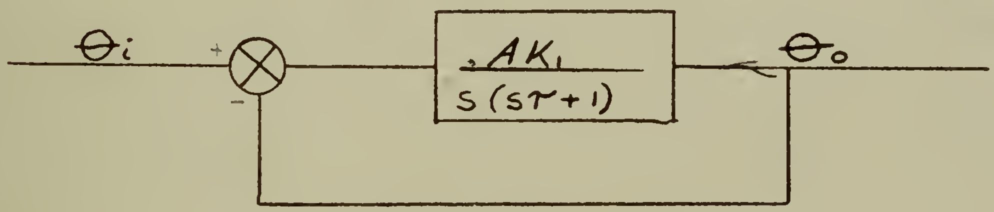


Fig. 1.1

Fig. 1.2 portrays a typical transient response to a step input of the servo system shown in Fig. 1.1.

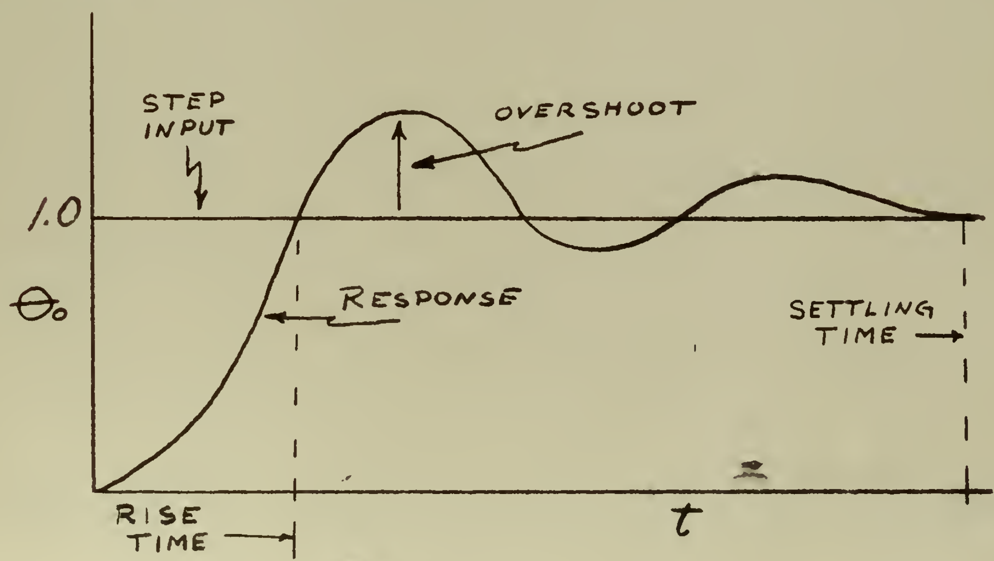


Fig. 1.2

It is obvious that during the initial portion of the response it in no way resembles the step input that was applied to the system. In order to improve the response two main objectives must be achieved. First, the rise time must approach zero and second, the oscillations must be damped out. The first objective can be realized simply by increasing the forward loop gain K_1. The response now will be as shown in Fig. 1.3.

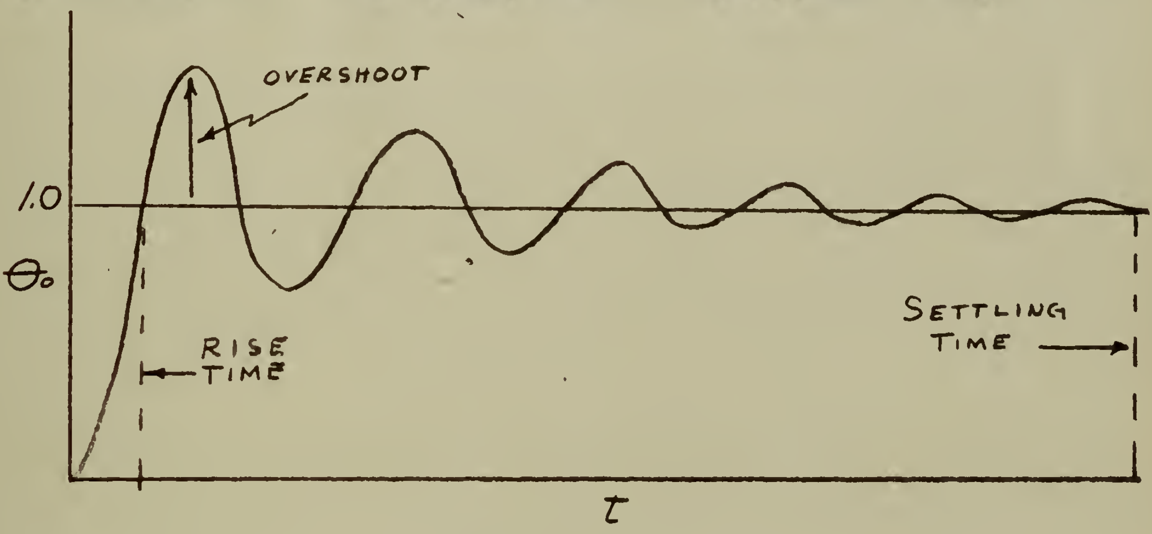


Fig. 1.3

The first objective has been achieved but in so doing the overshoot has been increased and the settling time has been increased due to the increased number of oscillations.

The second objective can be achieved by adding some means of compensation to the system. Fig. 1.4 shows the system with a tachometer feedback circuit added.

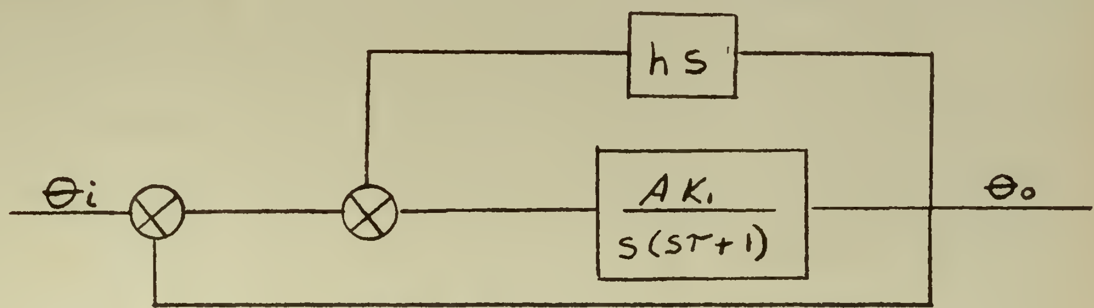


Fig. 1.4

The amplification in the tach channel (h) can be adjusted until the system is heavily overdamped. The response is now as shown in Fig. 1.5.

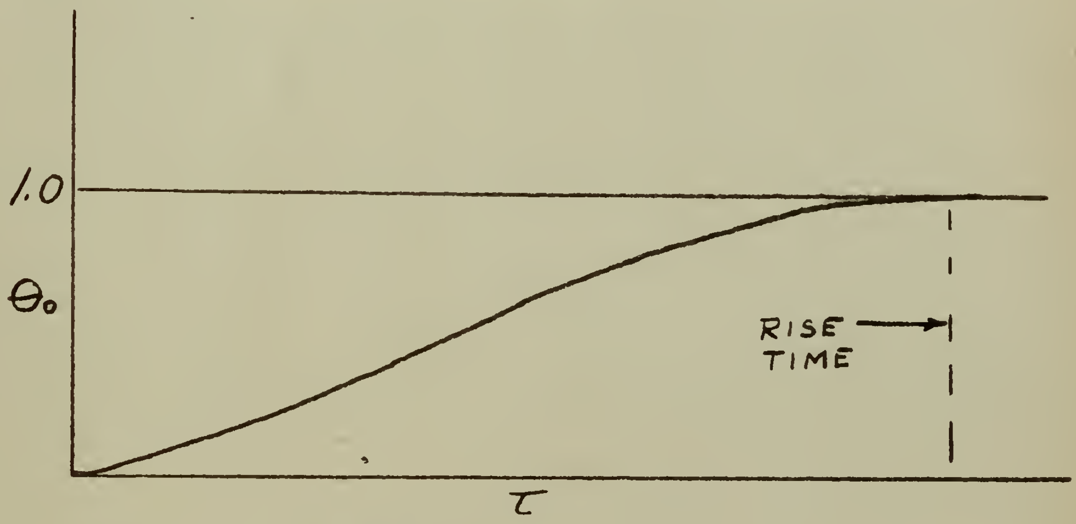


Fig. 1.5

The addition of the tachometer compensator has successfully achieved the second objective of damping out all the oscillations but in so doing has extended the rise time considerably.

The optimum system (one which will duplicate the input signal) then, is one which must possess the advantages of both the above cases but none of the disadvantages.

C) CONCEPT OF DELAYED COMPENSATION

It will be noted that the response of Fig. 1.2 is desirable only during the rise time and that the response of Fig. 1.5 is desirable only after the rise time. Herein lies the basic fundamentals of the concept of delayed compensation.

The proposed optimum system must therefore in effect consist of two separate systems. First, one which is underdamped to a point of instability. A system of this sort necessarily has a very small damping factor (ζ_1) and has a very short rise time. As the response of the

system nears the desired steady state value, the switch in the tach channel of Fig. 1.6 is closed producing a second system with a very high damping ratio (ζ_2).

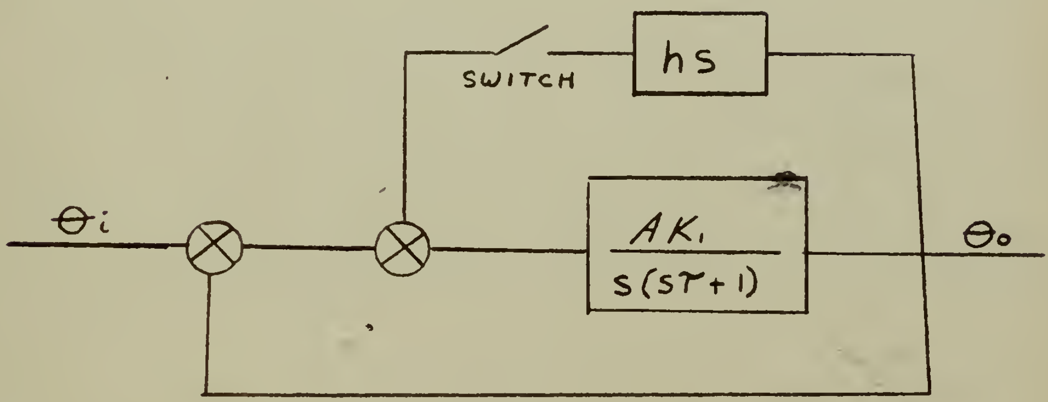


Fig. 1.6

An intuitive analysis of the system of this sort will produce a response such as that shown in Fig. 1.7.

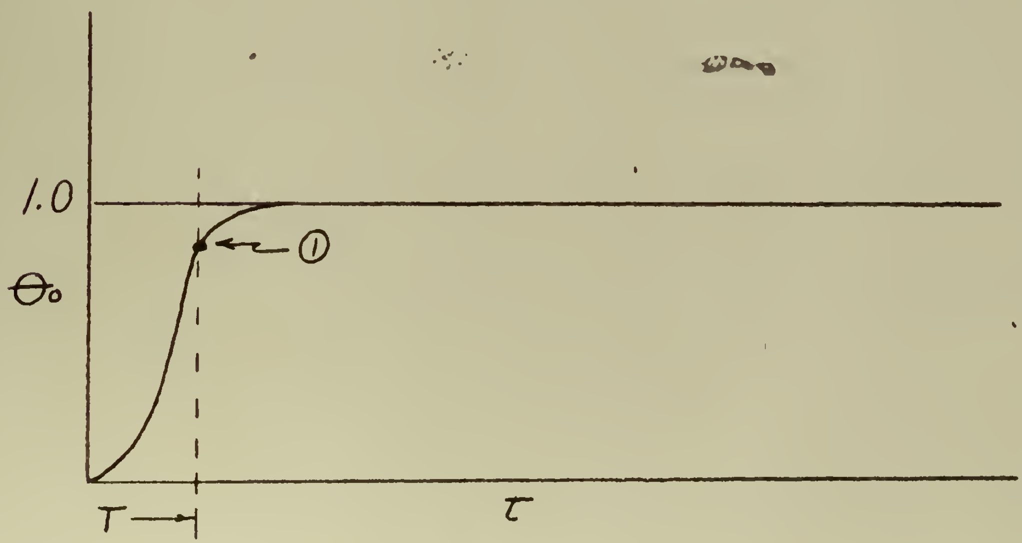


Fig. 1.7

As explained before, the early portion of the response is a result of a highly underdamped system. At point 1, after a time lapse of T, the switch in the tach channel is closed which increases the damping and converts the original underdamped system to a highly overdamped system.

The optimum response is approached when the system has a maximum rate of climb during the period T, and after t = T the response is flat as possible. This can be achieved by designing the system such that the damping ratio before the introduction of the tach feedback (ζ_1) approaches zero and the damping ratio after the introduction of the tach feedback approaches infinity.

Up to this point it has been assumed that the optimum response is the most desirable response. This is true only within certain limits. The practicality of the optimum response will be treated in more detail in Chapt. IV.

The foregoing concept has ignored the possibility that a transient may exist as the system adjusts itself from the underdamped case to the overdamped case. If the computer solutions in the following chapters do not indicate any such transient, it will be assumed that, if it does exist, its magnitude is negligible and will be ignored. However, if an appreciable transient is obvious in the computer analysis, then steps will be taken to either eliminate or reduce the effect of the transient.

This paper will consider both second and third order systems compensated by tachometer feedback and lead networks.

II SECOND ORDER SYSTEMS

A) TACHOMETER FEEDBACK COMPENSATION

1) MATHEMATICAL ANALYSIS

Derivation of ζ_1 (damping ratio prior to the introduction of the tach feedback)

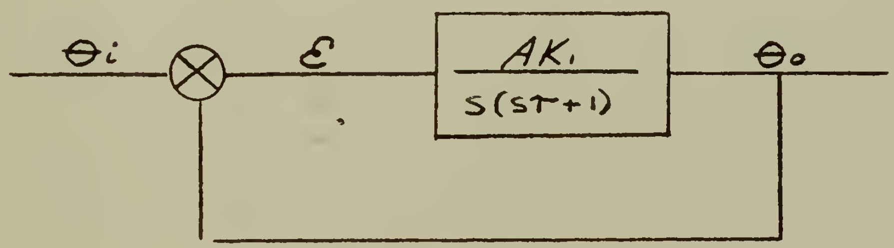


Fig. 2.1

$$\theta_o/\theta_i = \frac{G}{1+GH}$$

$$= \frac{\frac{AK_1}{s(s\tau+1)}}{1+\frac{AK_1}{s(s\tau+1)}}$$

$$= \frac{\frac{AK_1}{\tau}}{s^2+\frac{1}{\tau}s+\frac{AK_1}{\tau}}$$

converting to general notation

$$\frac{\theta_o}{\theta_i} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

WHERE

$$\omega_n^2 = \frac{AK_1}{\tau}$$

$$2\zeta\omega_n = \frac{1}{\tau}$$

$$2.1)\quad \zeta_1 = \frac{1}{2\tau\sqrt{AK_1/\tau}} = \frac{1}{2\sqrt{AK_1\tau}}$$

Derivation of ζ_2 (damping ratio after introduction of tach feed-back).

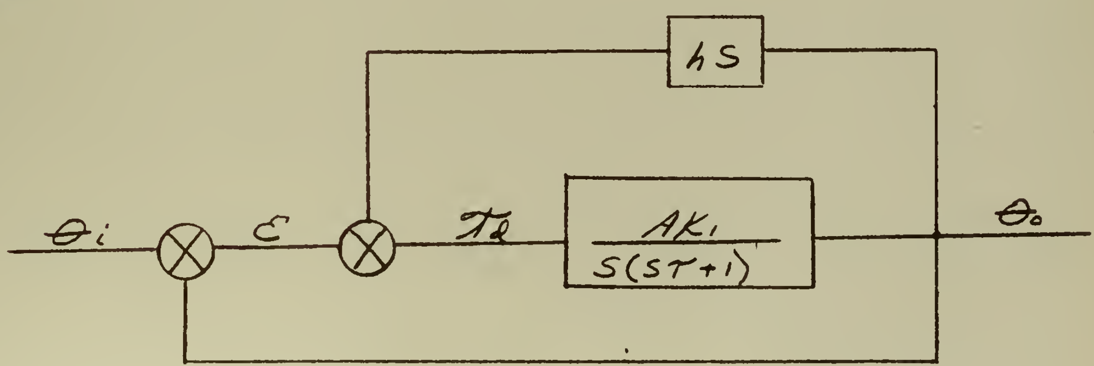


Fig. 2.2

$$\Theta_o/\mathcal{E} = \frac{\dfrac{AK_1}{s(s\tau+1)}}{1 + \dfrac{AK_1hs}{s(s\tau+1)}} = \frac{AK_1}{s(s\tau+1)+AK_1hs}$$

$$= \frac{\dfrac{AK_1}{\tau}}{s^2 + \left(\dfrac{AK_1h+1}{\tau}\right)s}$$

$$\Theta_o/\Theta_i = \frac{\dfrac{AK_1}{\tau}}{s^2 + \left(\dfrac{AK_1h+1}{\tau}\right)s + \dfrac{AK_1}{\tau}}$$

$$\omega_n^2 = \frac{AK_1}{\tau}$$

$$2\zeta\omega_n = \frac{AK_1h+1}{\tau}$$

2.2) $$\zeta_2 = \frac{AK_1h+1}{2\tau\sqrt{AK_1/\tau}} = \frac{AK_1h+1}{2\sqrt{AK_1\tau}}$$

2) COMPUTER STUDY

This section is devoted to a study of the effects of delayed compensation by means of an analog computer. The computer used in this thesis was a Donner analog computer and associated equipment.

It is assumed in this section that a particular system exists and it is desired to determine the effects of the variable parameters. In other words, the values inherent to the system, A + τ are fixed and the variable parameters K_1, h, + T will be altered and the effects noted graphically. Varying the parameters K_1 + h will effect the damping ratios (see Eq'n. 2.1 and 2.2) and varying T merely changes the duration of time in which the underdamped system is allowed to operate.

The block diagram shown in Fig. 1.6 is altered slightly in order to adapt the system to analog computer analysis. The computer study will consider the system as it is shown in Fig. 2.3.

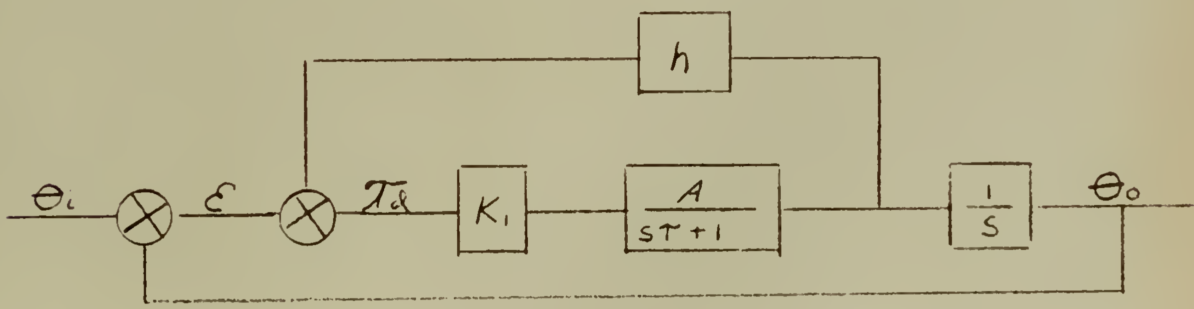


Fig. 2.3

The above diagram cannot be realized physically but it is mathematically identical to the diagram in Fig. 1.6. This arrangement is employed in the computer analysis because it precludes the necessity of differentiating in the tachometer channel.

A block diagram of the computer set up is shown in Fig. 2.4

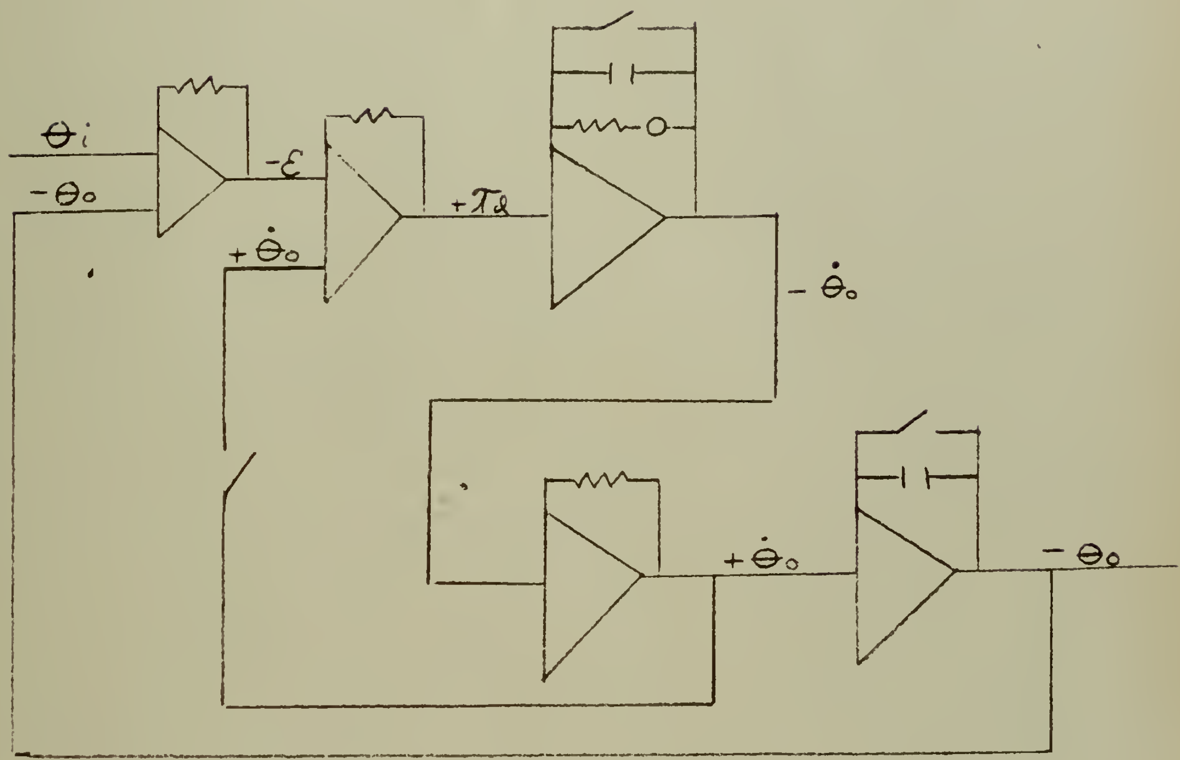


Fig. 2.4

As mentioned before the parameters that are varied in the computer study are ζ_1 ζ_2 and τ. Fig. 2.5, 2.6 and 2.7 show the transient response of the system. Each figure maintains a constant $\zeta_1 + \zeta_2$ and indicates the effect of a variable τ. The three figures indicate the interdependence of the three variables.

For each value of ζ_1 there exists an infinite pair of values for τ and ζ_2, That will bring the system into correspondence. If a deadbeat response is desired, however, ζ_2 should approach infinity and for any

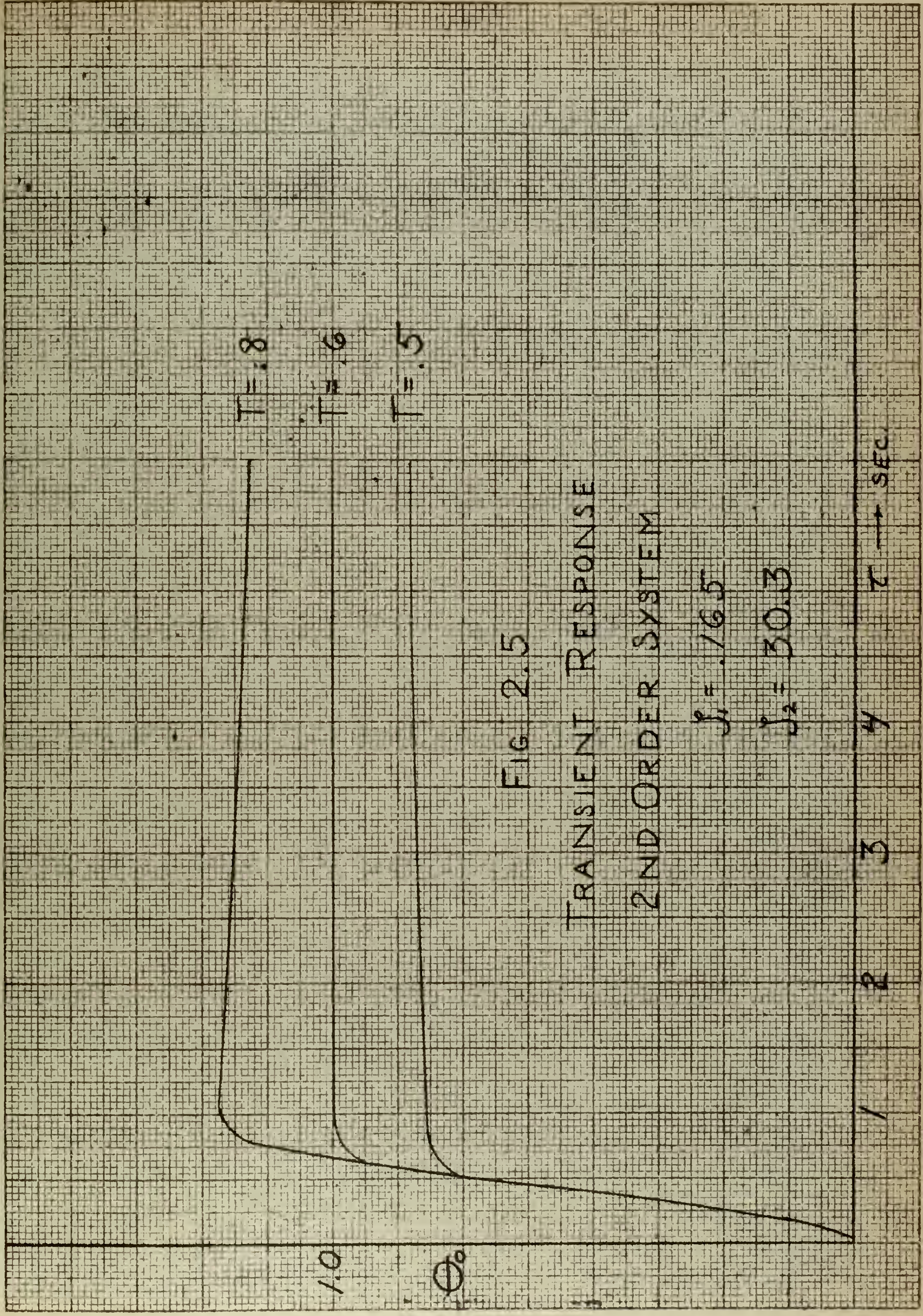


Fig. 2.5
Transient Response
2nd Order System
$f_1 = .165$
$f_2 = 30.3$
T = .8
T = .6
T = .5
1.0
θ_0
1
2
3
4
$\tau \longrightarrow$ sec.

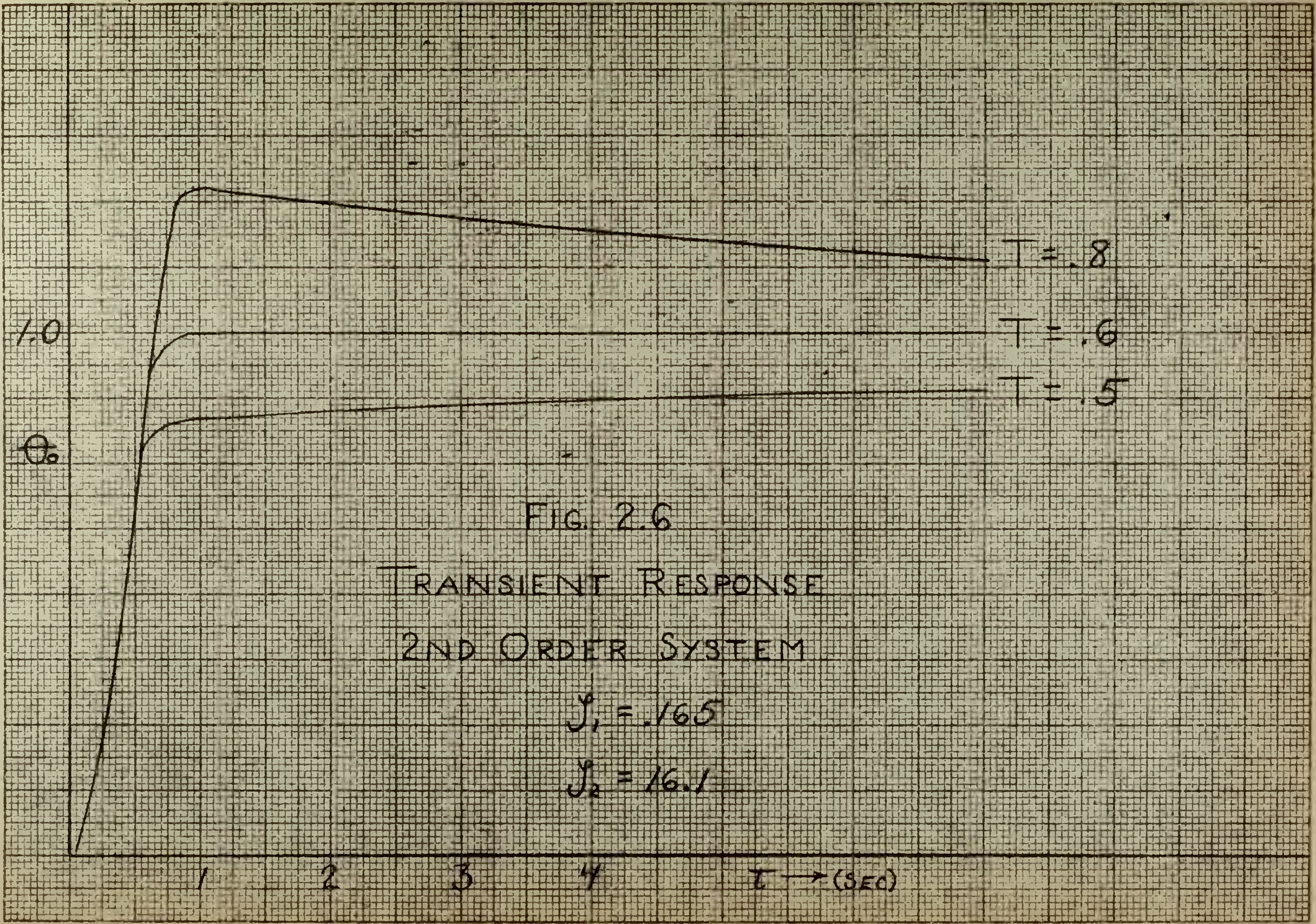


FIG. 2.6

TRANSIENT RESPONSE

2ND ORDER SYSTEM

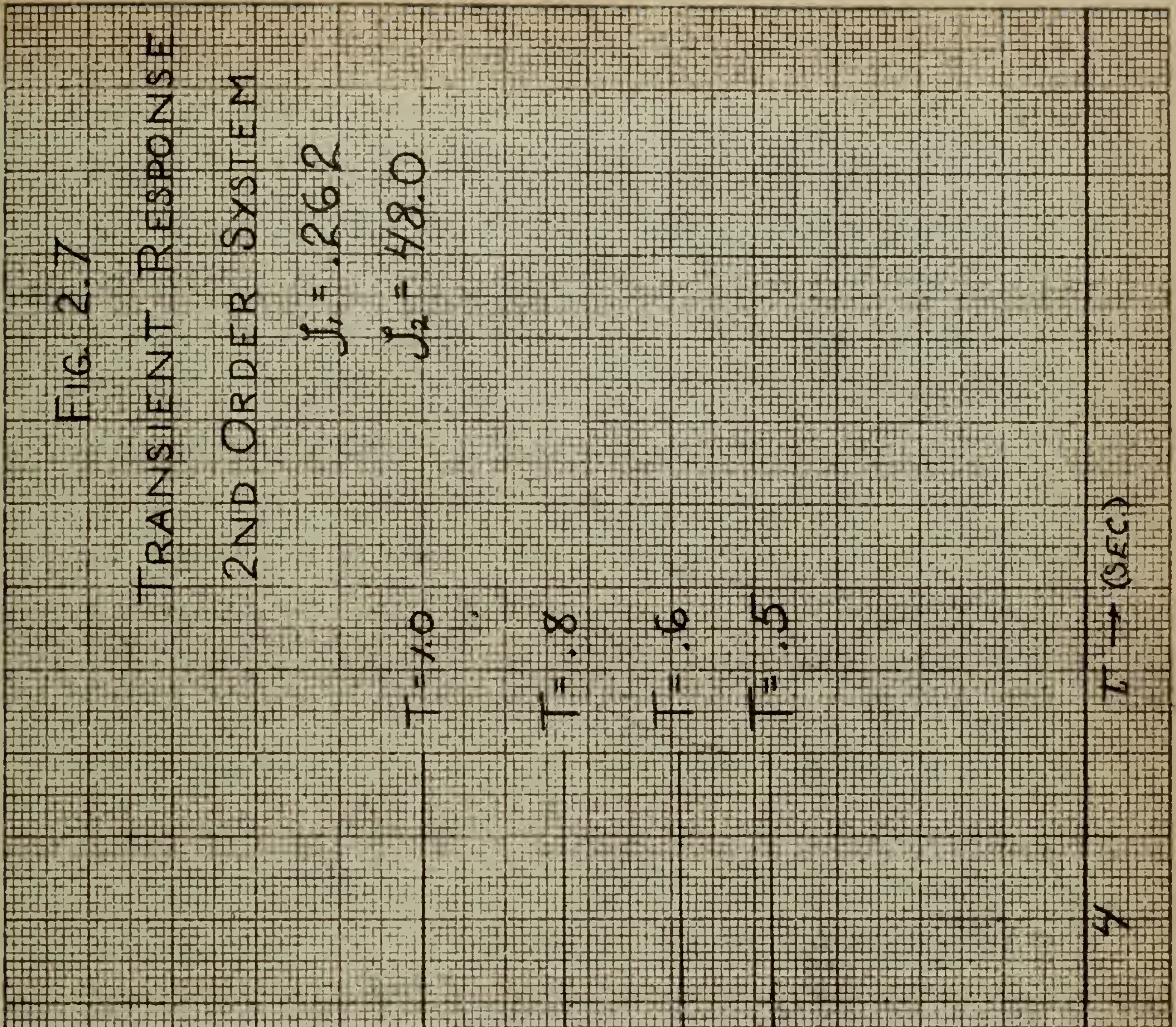
$J_1 = .165$

$J_2 = 16.1$

Fig. 2.7

Transient Response

2nd Order System



given high value of ζ_2 there exists only one value of T that will produce the deadbeat response. Likewise, for any value of ζ_2 there exists an infinite pair of values for T and ζ_1 that will bring the system into correspondence. Again, if a deadbeat response is desired, ζ_1 should approach zero and for any given low value of ζ_1 there exists only one value of T that will produce the deadbeat response. A similar relationship holds for a given value of T.

It will also be noted that a change in either ζ_1 or T has a much greater effect on the response than does a change in ζ_2.

Fig. 2.5, 2.6 and 2.7 show that at the instant of switching the system immediately assumes the response of an overdamped system. This indicates that if an intermediate transient exists during, or shortly after switching, its effects are negligible and can be ignored.

3) DESIGN CONSIDERATIONS

It is the purpose of this section to investigate the design problems encountered in adapting this method of control to an existing second order system.

a) GENERAL

It is obvious that the success of this method of control is dependent upon the ability of the hardware to respond to a large negative voltage after switching occurs in the tach channel and thereby produce a sufficient negative torque that will rapidly bring the system into correspondence. The "required torque" characteristics will therefore be used as a basis of analysis. The "required torque" is defined as that torque which the system must be able to produce in order to bring the response into correspondence rapidly. The "capable torque" is defined as that maximum

torque that the system is capable of producing. The fundamental design problem will be to adapt this method of control to an existing system so that the required torque is equal to or less than the capable torque.

b) DESIGN CRITERIA

Up to this point it was assumed that the torque developed by a motor is directly proportional to the applied armature voltage. This is a valid assumption, BUT ONLY WITHIN A FINITE RANGE! If an excessive voltage is applied to the motor the torque begins to saturate and is no longer directly proportional to the applied voltage. Fig. 2.8 portrays a typical torque vs applied armature voltage characteristics.

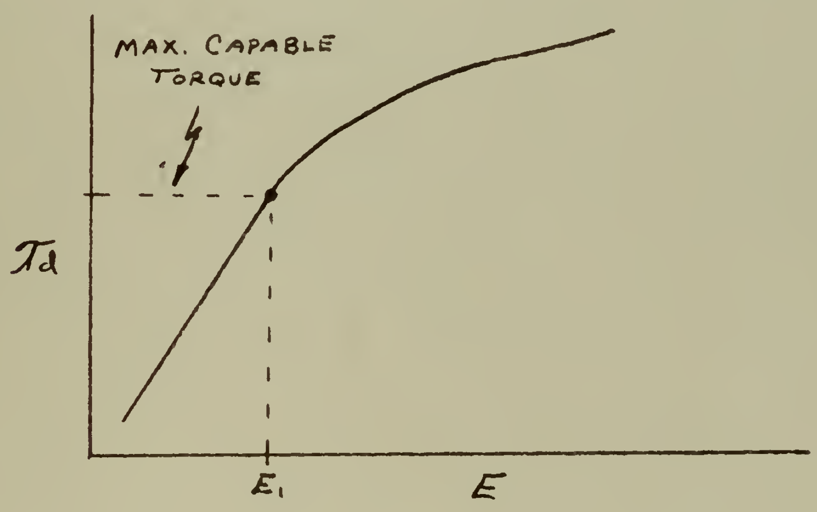


Fig. 2.8

In Fig. 2.8 the torque is linear only in the range $0 < E < E_1$. In designing the system therefore, it is necessary to insure that no torque in excess of the capable torque is demanded. The capable torque of the motor will therefore be the basis for the design considerations.

Fig. 2.9 is a typical representation of the nature of the torque

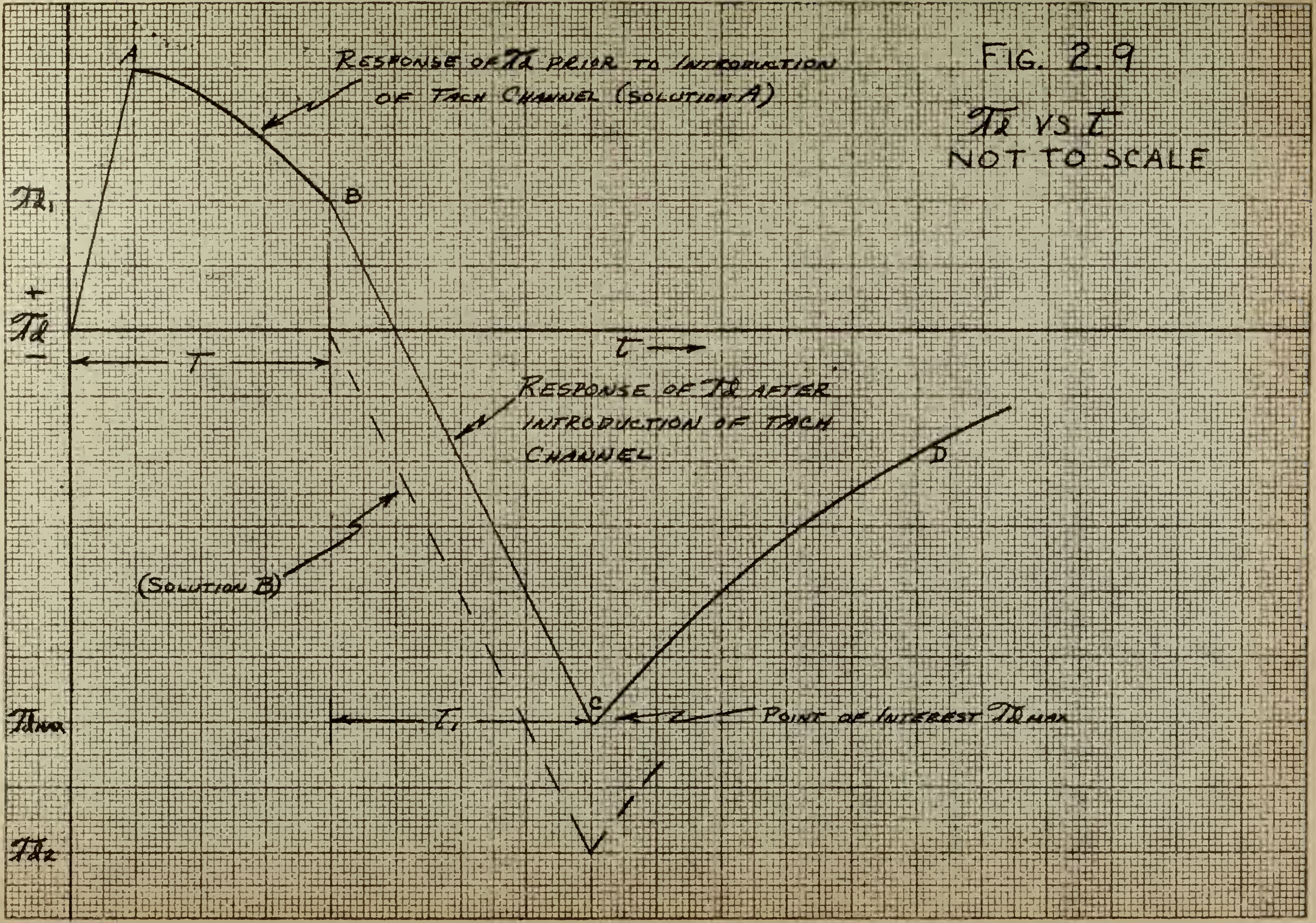


FIG. 2.9
VS t
NOT TO SCALE
RESPONSE OF PRIOR TO INTRODUCTION OF TACH CHANNEL (SOLUTION A)
RESPONSE OF AFTER INTRODUCTION OF TACH CHANNEL
(SOLUTION B)
POINT OF INTEREST MAX
A
B
C
D
T
T_1
t
+
−

response of a system employing delayed tachometer feedback compensation. The graph is not to scale and has been expanded along the time axis for clearer presentation.

Referring to Fig. 2.9, the initial rapid rise of torque (OA) is due to the introduction of the step input. The response (AB) is the normal torque response prior to the introduction of the compensation. At point B the tach feedback switch is closed which rapidly reverses the direction of the torque (BC). Point C is the maximum required torque that the motor must be capable of producing. In designing a system this maximum required torque must be known so that a motor of sufficient size can be employed to insure linear operation. If this method of control is being adapted to an existing system it is necessary to express the maximum required torque in terms of the parameters of the system so that the parameters can be adjusted until the maximum required torque is compatible with the maximum capable torque of the existing motor.

The following two sections show the procedure for determining this maximum torque in terms of the system parameters.

c) OUTLINE OF PROCEDURE

This section is a descriptive outline for the procedure for analyzing the torque in terms of system parameters. The actual analysis is contained in the following section.

Derive the mathematical expression for the torque response for solution A, (see Fig. 2.9), from this determine the value of the torque at $\tau = T_1$. This value of torque will be referred to as $\mathcal{T}_{Q_1}$.

Derive the mathematical expression for the feedback voltage (even though it does not affect the system during the time of solution A) and

determine its value at $t = T$. This value of feedback voltage will be referred to as M.

Inasmuch as $\mathcal{T}_{d\,MAX}$ is the only information that is desired from this analysis, the following assumptions can be made:

1) When $t = T$, θ_i is constant and is equal to B in magnitude.

2) Upon closing the tach channel switch the feedback voltage is assumed to be a step function momentarily an equal in magnitude to M.

At the instant the switch closes assume a new problem exists. Shift the $\mathcal{T}_d$ axis such that $\mathcal{T}_{d_1} = 0$. Assume $\theta_i = B$ and that the forcing function is now the feedback voltage and is equal to M/S . Now derive the expression for solution B . Employ the initial value theorem to determine $\mathcal{T}_{d_2}$. Now subtract $\mathcal{T}_{d_1}$ from $\mathcal{T}_{d_2}$ to get the actual value of $\mathcal{T}_{d\,MAX}$.

The expression for the maximum required torque will now be in terms of $A, \tau, K_1, h, B + T$. For any given system $A + \tau$ will be existing constants. This leaves the expression for $\mathcal{T}_{d\,MAX}$ in terms of four variables $T, B, h + K_1$.

The choice of T will depend mostly on the amount of overshoot, undershoot and time to correspondence that can be tolerated. For a step input T will be independent of the size of the input signal because of the linear nature of the initial rise of the response.

B (the size of the step input) will either be the same constant at all times or will vary in magnitude between certain limits depending on the application of the existing system.

In order to achieve an optimum response ζ_2 should be as large as possible and ζ_1 should be as small as possible as explained in Chapter I. This should be kept in mind when choosing the values of K_1 & h because:

$$\zeta_1 = \frac{1}{2\sqrt{AK_1\tau}} \quad \text{AND} \quad \zeta_2 = \frac{AK_1h+1}{2\sqrt{AK_1\tau}}$$

However, as Chapter V points out, it may not be desirable to make ζ_2 as large as possible for practical considerations.

d) ANALYSIS

(refer to Fig. 2.9)

SOLUTION A

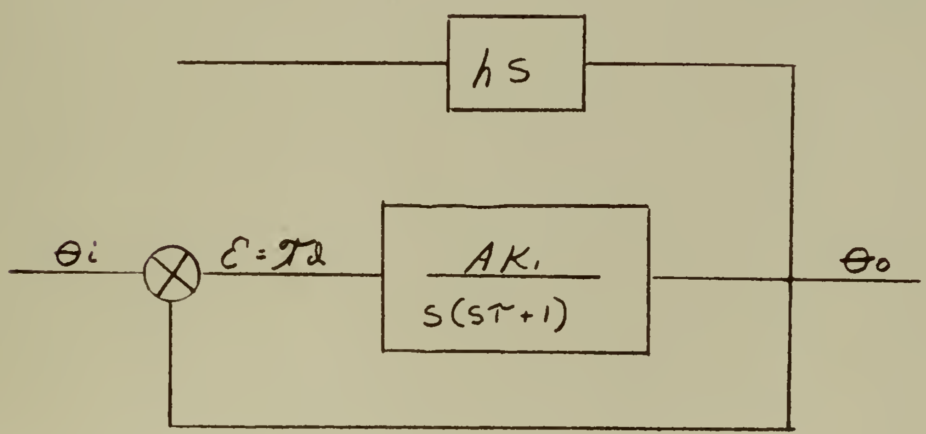


Fig. 2.10

$$T_d = \mathcal{E} = \theta_i - \theta_o$$

$$\theta_o = \frac{\frac{AK_1}{s(s\tau+1)}}{1+\frac{AK_1}{s(s\tau+1)}}\,\theta_i = \frac{AK_1}{s(s\tau+1)+AK_1}\,\theta_i$$

$$\theta_o = \frac{\frac{AK_1}{\tau}}{s^2+\frac{1}{\tau}s+\frac{AK_1}{\tau}}\,\theta_i$$

$$T_d = \left[1 - \frac{\frac{AK_1}{\tau}}{s^2+\frac{1}{\tau}s+\frac{AK_1}{\tau}}\right]\theta_i$$

$$T_d = \frac{s(s+\frac{1}{\tau})}{s^2+\frac{1}{\tau}s+\frac{AK_1}{\tau}}\,\frac{B}{s} = \frac{B(s+\frac{1}{\tau})}{s^2+\frac{1}{\tau}s+\frac{AK_1}{\tau}}$$

SOLVING ABOVE EQ'N

2.1) $T_d = f_1(B, A, K_1, \tau, t)$

SUBSTITUTING $t = T$

2.2) $T_{d1} = f_1(B, A, K_1, \tau, T)$

Now determine magnitude of feedback voltage at t = T

$$h s \theta_o = \frac{h s \frac{AK_1}{\tau}}{s^2 + \frac{1}{\tau} s + \frac{AK_1}{\tau}} \theta_i$$

$$= \frac{h B \frac{AK_1}{\tau}}{s^2 + \frac{1}{\tau} s + \frac{AK_1}{\tau}}$$

2.3) $h s \theta_o = f_2 (A, h, B, \tau, K_1, t)$

SUBSTITUTING $t = T$

2.4) $h s \theta_{o\ \text{AT}\ t=T} = M = f_2 (A, h, B, \tau, K_1, T)$

SOLUTION B

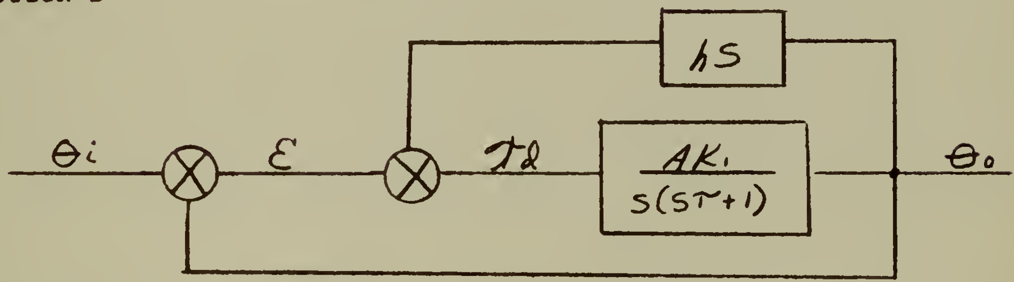


Fig. 2.11

Now assume $T_d = 0$ at the instant the tach feedback channel is closed (amounts to shifting the T_d axis.)

$$T_d = \mathcal{E} - h s \theta_o$$

$$\mathcal{E} = \theta_i - \theta_o$$

$$T_d = B - \theta_o - h s \theta_o$$ (BECAUSE NOW θ_i IS ASSUMED TO BE CONSTANT)

$$T_d = B - \left(\frac{1}{hs} + 1\right) h s \theta_o$$

$$h s \theta_o = \frac{M}{s}$$ (ASSUMED TO BE A STEP FUNCTION AT $t = T$ OF MAGNITUDE M)

$$2.5)\quad \tau_d = B - \left(\frac{1}{hs} + 1\right)\frac{M}{s}$$

APPLYING THE INITIAL VALUE THEOREM

$$\tau_{d\,(t=0)} = s\left[F(s) - F(\infty)\right]_{s\to\infty}$$

$$\tau_{d_2} = s\left[B - \left(\frac{1}{hs} + 1\right)\frac{M}{s} - B\right]_{s\to\infty}$$

$$= s\left[-\frac{M}{hs^2} - \frac{M}{s}\right]_{s\to\infty}$$

$$= \left[-\frac{M}{hs} - M\right]_{s\to\infty}$$

$$2.6)\quad \tau_{d_2} = -M$$

NOW $\tau_{d\,MAX}$ IS FOUND BY

$$2.7)\quad \tau_{d\,MAX} = |\tau_{d_2}| - |\tau_{d_1}|$$

It is now obvious that it is only necessary to determine the actual form of equations 2.2 and 2.4 in order to express $\mathcal{T}\mathcal{Q}_{MAX}$ in terms of the system parameters.

Solving for 2.2

$$\mathcal{T}\mathcal{Q}_1 = \frac{B\left(S+\frac{1}{T}\right)}{S^2+\frac{1}{T}S+AK_1/T}$$

GENERAL FORM

$$= \frac{B(S+Z)}{S^2+2\alpha S+\omega_m^2}$$

WHERE

$$Z = \frac{1}{T} \qquad \omega_m^2 = \frac{AK_1}{T}$$

$$\alpha = \frac{1}{2T} \qquad \omega_c = \sqrt{\omega_m^2-\alpha^2} = \sqrt{\frac{AK_1}{T}-\frac{1}{4T^2}}$$

GENERAL SOLUTION BY RESIDUES

$$\mathcal{T}\mathcal{Q}_1 = \frac{BD\epsilon^{j\theta}\epsilon^{\alpha}}{\omega_c}\left[\frac{\epsilon^{j\omega_c\tau}}{2j} + \frac{\epsilon^{-j\omega_c\tau}}{-2j}\right]$$

WHERE

$$\theta = TAN^{-1}\frac{\omega_c}{-\alpha+Z}$$

$$D = -\alpha+Z$$

$$\mathcal{T}\mathcal{Q}_1 = \frac{BD\epsilon^{-\alpha\tau}}{\omega_c} \; SIN\left(\omega_c\tau+\theta\right)$$

THEREFORE

$$2.1)\quad \mathcal{T}\mathcal{Q}_1 = \frac{B\epsilon^{-\frac{1}{2T}\tau}}{2\sqrt{AK_1T-1/4}} \; SIN\left(\sqrt{\frac{AK_1}{T}-\frac{1}{4T^2}}\,\tau + TAN^{-1}2T\sqrt{\frac{AK_1}{T}-\frac{1}{4T^2}}\right)$$

SUBSTITUTING $t = T$

$$2.2)\quad \mathcal{L}\theta_1 = \frac{B\,\epsilon^{-\frac{1}{2\tau}T}}{2\sqrt{AK_1\tau - 1/4}}\ \sin\left(\sqrt{\frac{AK_1}{\tau} - \frac{1}{4\tau^2}}\,T + \tan^{-1} 2\tau\sqrt{\frac{AK_1}{\tau} - \frac{1}{4\tau^2}}\right)$$

SOLVING FOR EQ'N 2.4

$$\mathcal{L}\theta_2 = M = \frac{hB\,\frac{AK_1}{\tau}}{S^2 + \frac{1}{\tau}S + \frac{AK_1}{\tau}}$$

GENERAL FORM

$$= \frac{hB\,\omega_n^2}{S^2 + 2\alpha S + \omega_n^2}$$

WHERE

$$\omega_n^2 = \frac{AK_1}{\tau}$$

$$\alpha = \frac{1}{2\tau}$$

$$\omega_c = \sqrt{\omega_n^2 - \alpha^2} = \sqrt{\frac{AK_1}{\tau} - \frac{1}{4\tau^2}}$$

GENERAL SOLUTION

$$M = hB\,\omega_n^2\left[\frac{\epsilon^{-\alpha t}}{\omega_c}\sin\omega_c t\right]$$

$$2.3)\quad M = \frac{hBAK_1}{\tau}\left[\frac{\epsilon^{-\frac{1}{2\tau}t}}{\sqrt{\frac{AK_1}{\tau} - \frac{1}{4\tau^2}}}\ \sin\sqrt{\frac{AK_1}{\tau} - \frac{1}{4\tau^2}}\ t\right]$$

SUBSTITUTING $\tau = T$

$$2.4) \quad M = \frac{hBAK_1 \, \epsilon^{-\frac{1}{2\tau}T}}{\sqrt{AK_1\tau - 1/4}} \sin \sqrt{\frac{AK_1}{\tau} - \frac{1}{4\tau^2}}\, T$$

EQ'N 2.7 THEN BECOMES

$$2.7) \quad \mathcal{T}_{d\,MAX} = \frac{B\,\epsilon^{-\frac{1}{2\tau}T}}{\sqrt{AK_1\tau - 1/4}} \left[hAK_1 \sin\theta T - \frac{1}{2}\sin\left(\theta T - \tan^{-1} 2\tau\theta\right) \right]$$

WHERE

$$\theta = \sqrt{\frac{AK_1}{\tau} - \frac{1}{4\tau^2}}$$

ILLUSTRATIVE EXAMPLE:

PROBLEM - to adapt this method of control to an existing system with the following constants:

$$A = .914 \qquad T = 1$$

Capable maximum linear to4que = 2.67

Design specifications

1) $0 < B \leq 1$ (in analysis use maximum value B = 1)

2) ζ_2 shall not be as high as possible as in the optimum case. but shall be limited to 2 for acceleration and control reasons.

3) Steady state shall be attained prior to t = 1.0 seconds.

4) For practical reasons h < 15, K < 15

DESIGN

$$\zeta_2 = \frac{1 + .914 h K_1}{2\sqrt{.914 K_1}} = 2$$

$$.914 h K_1 = 4\sqrt{.914 K_1} - 1$$

$$h = \frac{4\sqrt{.914}\sqrt{K_1} - 1}{.914 K_1} = \frac{4.18}{\sqrt{K_1}} - \frac{1.1}{K_1}$$

EQ'N 2.7 REDUCES TO

$$2.67 = \frac{\epsilon^{-\frac{T}{2}}}{\sqrt{.914K_1 - 1/4}}\left[(3.83\sqrt{K_1} - 1)\sin\theta T - 1/2\sin(\theta T - \tan^{-1}2\theta\right]$$

WHERE

$$\theta = \sqrt{.914K_1 - \frac{1}{4}}$$

to stay within the limits of specification (3) a logical first guess for T would be T = .6

$$2.67 = \frac{.741}{\sqrt{.914 K_1 - 1/4}} \left[(3.83\sqrt{K_1} - 1) \sin .6\theta - \tfrac{1}{2} \sin(.6\theta - \tan^{-1} 2\theta) \right]$$

SOLVING FOR K_1

$$K_1 = 10$$

THEN FOR h, ζ_1 & ζ_2

$$h = \frac{4.18}{\sqrt{10}} - \frac{1.1}{10} = 1.32 - .11 = 1.21$$

$$\zeta_1 = \frac{1}{2\sqrt{9.14}} = .161$$

$$\zeta_2 = \frac{1 + (9.14)(1.21)}{2\sqrt{9.14}}$$

$$= \frac{12.1}{6.04} \approx 2$$

e) EFFECT OF TORQUE SATURATION

Section IIA3b points out that the primary design criteria for a tachometer feedback system is assuring that no torque is demanded of a motor outside of its linear range.

Consider now a system that is actuated by step inputs whose magnitudes periodically is subjected to large inputs that exceed the definite range. The design question to answer here is: Should the parameters of the system be so designed to yield an optimum response only to the inputs that fall in the above definite range or should the design consider all possible magnitudes of the input? The former is more desirable from an economical viewpoint but the response to the few large inputs may be intolerable due to torque saturation. Therefore, before this question can be answered the effect of torque saturation on the response of a system must be studied.

Fig. 2.12 is a computer representation of the torque produced by the motor. By employing a diode limiter in the torque channel of the computer, the torque is limited (or saturated) at various percentages of its maximum value.

Fig. 2.13 shows the transient response of the system with the same degrees of torque saturation depicted in Fig. 2.12. Under ideal conditions (no saturation) the response has the desired deadbeat configuration. As the torque is limited an undesirable overshoot condition develops. The magnitude of the overshoot is directly dependent on and proportional to the degree of saturation.

If this overshoot can be tolerated periodically in the application of the system then the design can be based on the normal range of inputs. If the application of the system demands a deadbeat response at all times then the entire range of possible inputs must be considered.

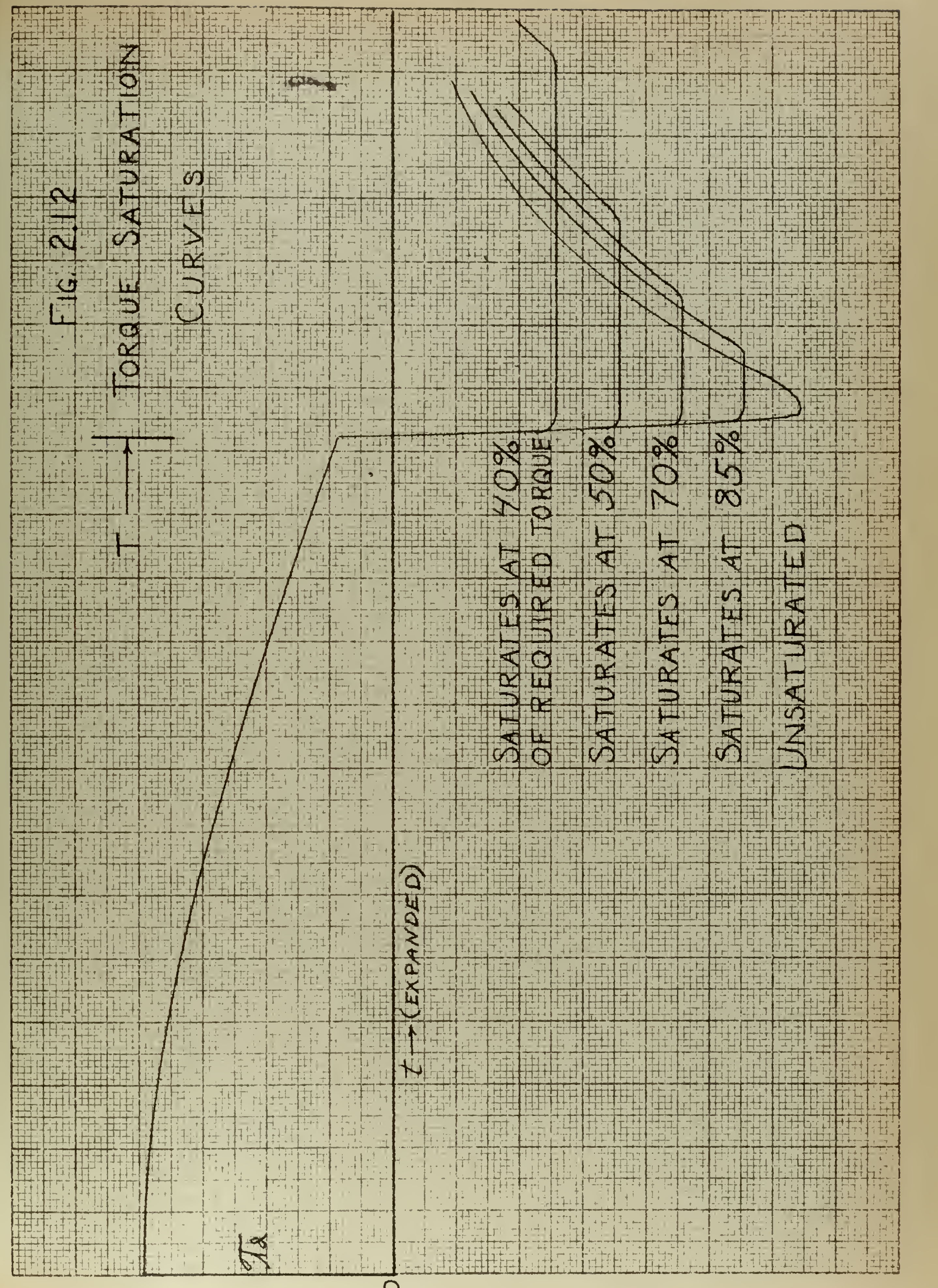


FIG. 2.12
TORQUE SATURATION CURVES
T
T_l
t → (EXPANDED)
O
SATURATES AT 40% OF REQUIRED TORQUE
SATURATES AT 50%
SATURATES AT 70%
SATURATES AT 85%
UNSATURATED

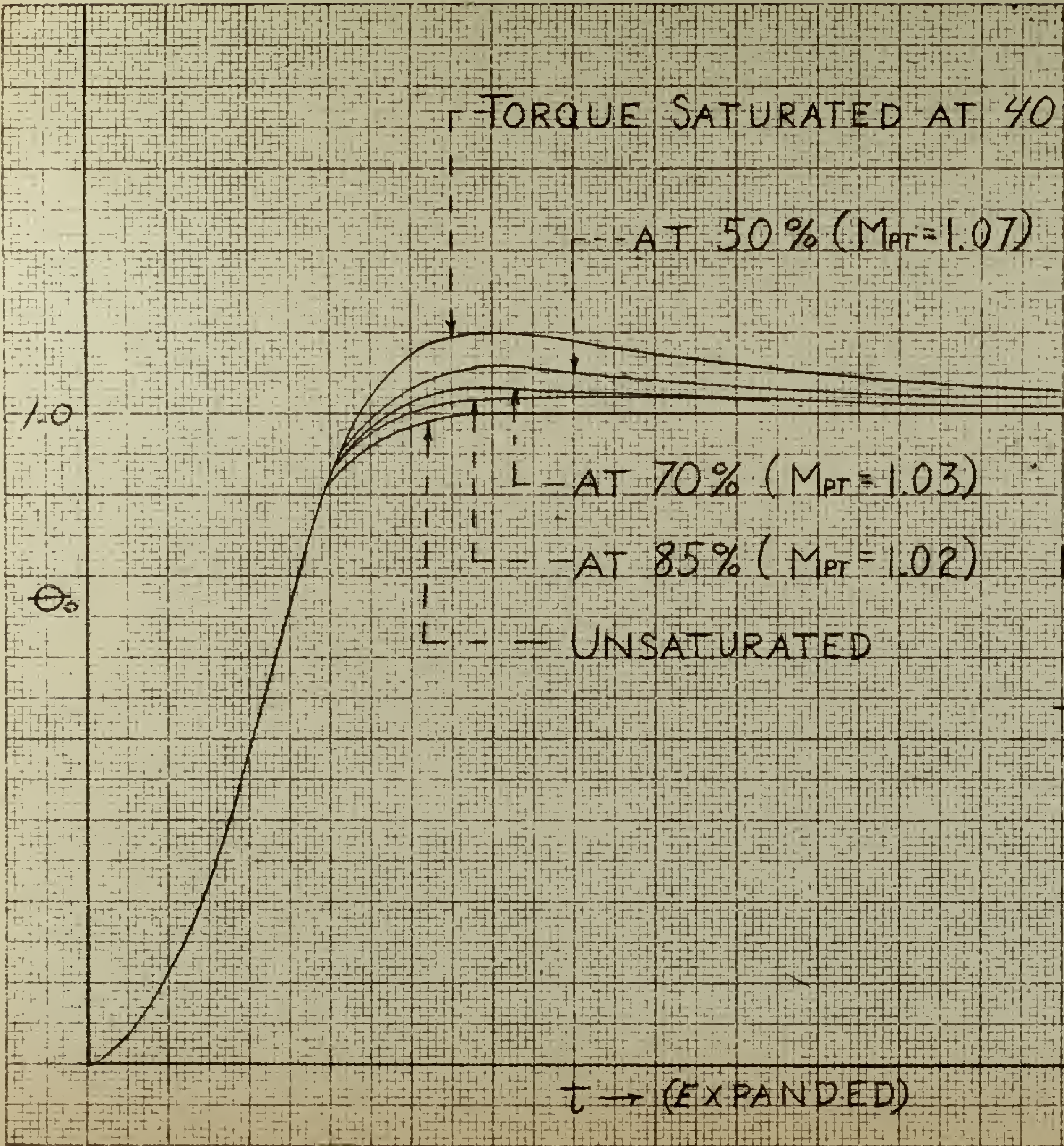


FIG. 2.13

EFFECT OF TORQUE SATURATION ON TRANSIENT RESPONSE

$\zeta_1 = .165$

$\zeta_2 = 2.0$

B) LEAD NETWORK COMPENSATION

1) MATHEMATICAL ANALYSIS

Derivation of ζ_1 (switch to position ①)

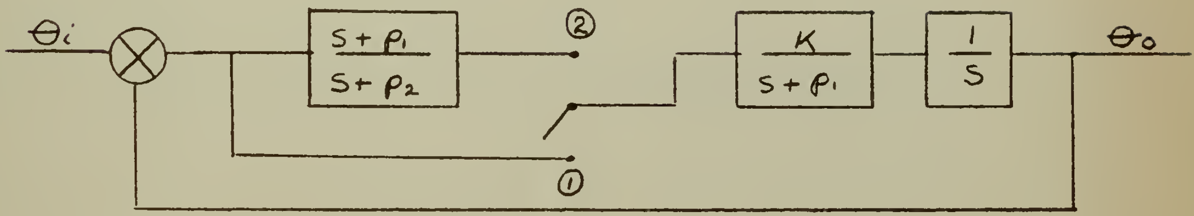


Fig. 2.14

$$\theta_o/\theta_i = \frac{\frac{K_1}{S(S+P_1)}}{1+\frac{K_1}{S(S+P_1)}} = \frac{K_1}{S(S+P_1)+K_1} = \frac{K_1}{S^2+P_1 S+K_1}$$

$$\omega_n^2 = K_1 \qquad \omega_n = \sqrt{K_1}$$

$$2\zeta_1 \omega_n = P_1$$

$$\zeta_1 = \frac{P_1}{2\omega_n} = \frac{P_1}{2\sqrt{K_1}}$$

LET $K_1 = 1$ $\quad P_1 = .333$

$$\zeta_1 = \frac{.333}{2} = .166$$

Derivation of ζ_2 (switch in position ②)

$$\frac{\theta_o}{\theta_i} = \frac{\frac{K_1}{S(S+P_2)}}{1+\frac{K_1}{S(S+P_2)}} = \frac{K_1}{S(S+P_2)+K_1} = \frac{K_1}{S^2+P_2 S+K_1}$$

$$\omega_n^2 = K_1 \qquad \omega_n = \sqrt{K_1}$$

$$2\zeta_2 \omega_m = \rho_2$$

$$\zeta_2 = \frac{\rho_2}{2\omega_m} = \frac{\rho_2}{2\sqrt{K_1}}$$

LET $\rho_2 = 4$

$$\zeta_2 = \frac{4}{2} = 2$$

General design of lead network.

Fig. 2.15 represents the circuitry for the lead network.

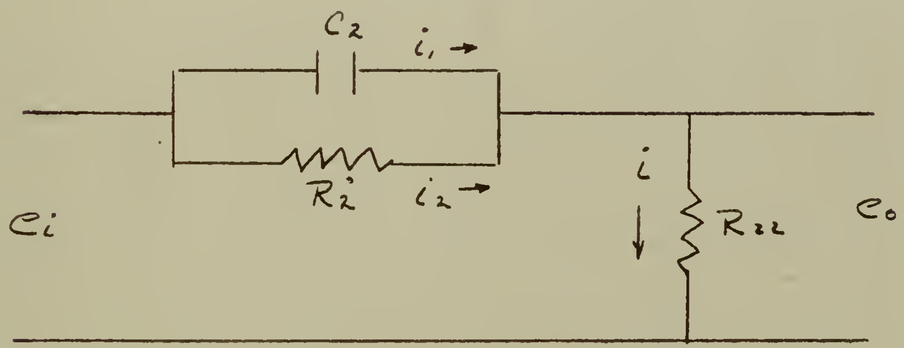


Fig. 2.15

$$i_1 + i_2 = i$$

$$i_1 \frac{1}{C_2 S} = i_2 R_2$$

$$e_o = i R_{22}$$

$$i = i_1\left(1 + \frac{1}{C_2 R_2 S}\right)$$

$$e_i = i_1 \frac{1}{C_2 S} + e_o$$

$$e_i - e_o = i_1 \frac{1}{C_2 S} = i \frac{1}{1 + \frac{1}{C_2 R_2 S}} \frac{1}{C_2 S} = i \frac{1}{C_2 S + \frac{1}{R_2}}$$

$$i = (e_i - e_o)\left(C_2 S + \frac{1}{R_2}\right)$$

$$e_o = (e_i - e_o)\left(C_2 S + \frac{1}{R_2}\right) R_{22}$$

$$e_o = e_i\left(C_2 R_{22} S + \frac{R_{22}}{R_2}\right) - e_o\left(C_2 R_{22} S + \frac{R_{22}}{R_2}\right)$$

$$e_o = \frac{C_2 R_{22} S + \frac{R_{22}}{R_2}}{C_2 R_{22} S + 1 + \frac{R_{22}}{R_2}} \; e_i$$

$$e_o = \frac{S + \frac{R_{22}}{R_{22} R_2 C_2}}{S + \frac{R_{22} + R_2}{R_{22} R_2 C_2}} \; e_i = \frac{S + P_1}{S + P_2} \; e_i$$

WHERE

$$\frac{1}{R_2 C_2} = .33$$

$$\frac{R_{22} + R_2}{R_{22} R_2 C_2} = 4$$

LET $C_2 = 1 \mu f$ AND KEEP $R_2 / R_{22} \approx 10$

$$\frac{1}{10 R_{22}} = .333$$

$$R_{22} = .3 \; M\Omega$$

$$\frac{11 R_{22}}{10 R_{22}^2} = \frac{1.1}{R_{22}} = \frac{1.1}{.3} = 3.67 \text{ (CLOSE TO 4)}$$

$$R_2 = 3.0 \; M\Omega$$

2) COMPUTER STUDY

Fig. 2.16 shows the computer set up for the block diagram in Fig. 2.14.

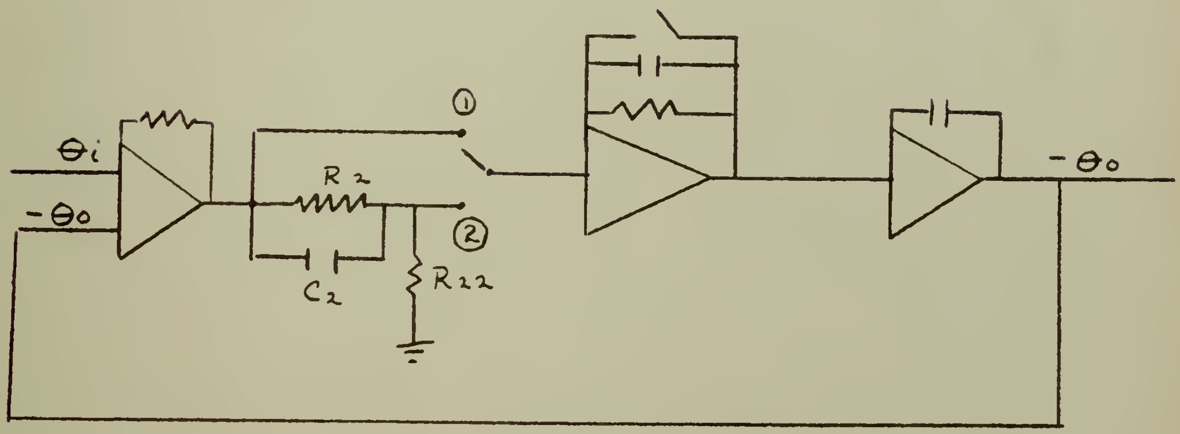


Fig. 2.16

Fig. 2.17 shows a family of curves obtained from the above set up. The computer values are such that $\zeta_1 = .166$ and $\zeta_2 = 2.0$. The switching time is varied from $T_2 = .1$ to $T = 2.0$.

It is quite obvious that the response of the lead network second order system does not compare favorably with the tachometer feedback second order system. In order to make the comparison with all other factors equal (damping ratios, switching time, etc.) refer to Fig. 4.1 in Chapter IV. Fig. 4.1 has identically the same damping ratios and considers practically the same switching time but the responses are much more desirable than those in Fig. 2.17. Notice in Fig. 2.17 that at the instant of switching there is no radical deviation in the response curve as there is in the tachometer feedback response. The response seems to react very sluggishly for a few seconds before it assumes the response of an over damped system.

This marked difference between the two systems seems to indicate that there is a sizeable transient occuring at the instant of switching in the case of the lead network. An investigation of Fig. 2.18, which is the lead net work portion of Fig. 2.16, points out the cause of this transient.

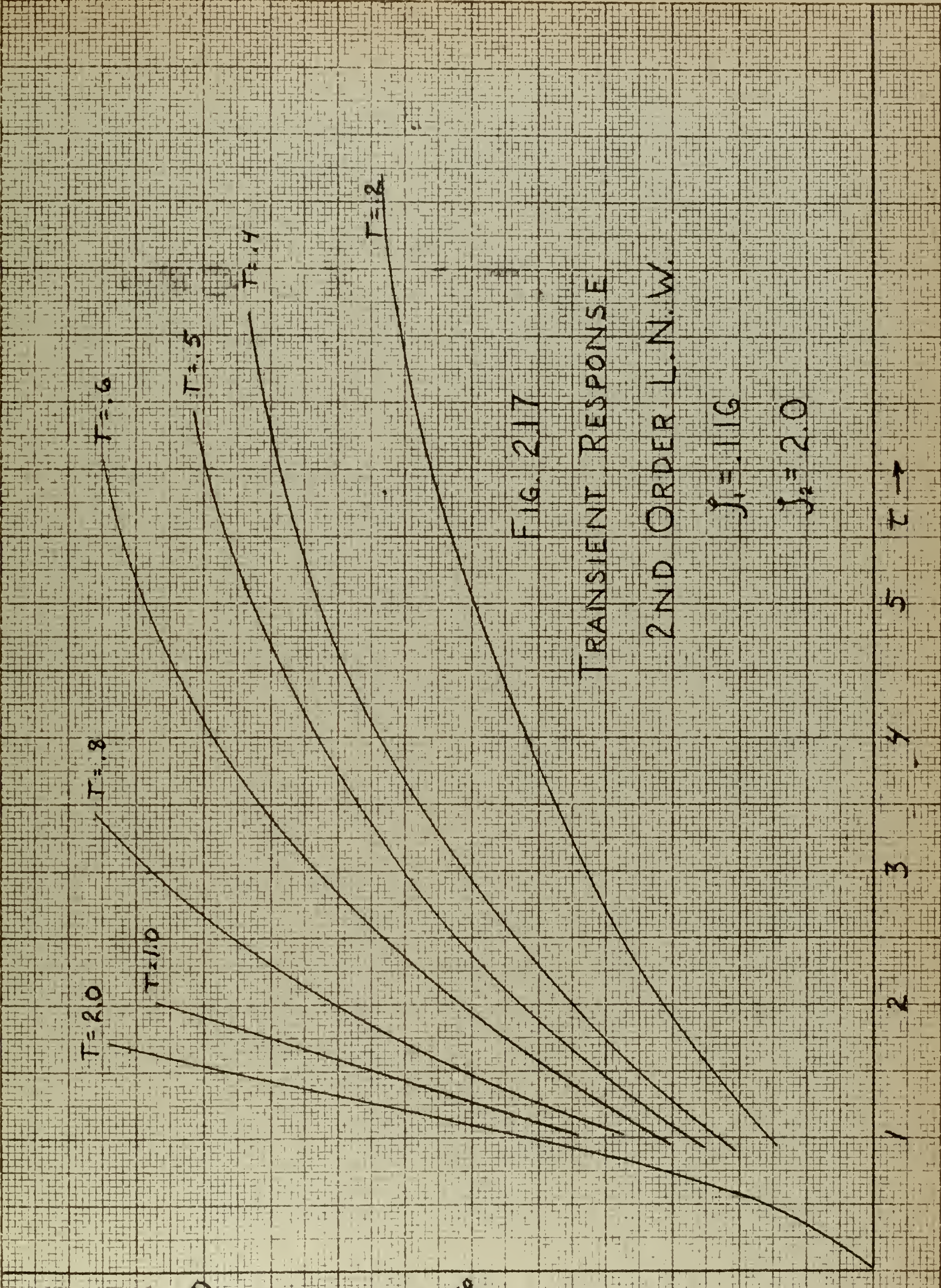


FIG. 2.17
TRANSIENT RESPONSE
2ND ORDER L. N. W.
$\zeta_1 = 1.16$
$\zeta_2 = 2.0$

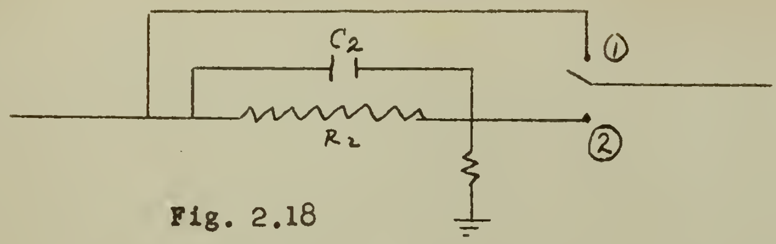


Fig. 2.18

During the initial phase of the response the switch is in position ① . The voltage at point ① is actually the error voltage and varies accordingly with time. While the switch is in position ① current is flowing thru the lead network to ground. In so doing point ② assumes a voltage that varies with time and is dependent not only on the error voltage but also on the values of C_2, R_2 & R_{22} . At the instant of switching the voltages at points ① and ② will be different. The switching action therefore introduces an initial condition into the system which was not considered in the mathematical analysis. This initial condition is the agency responsible for the adverse transient that causes the sluggish response shown in Fig. 2.17. This paper will not attempt to mathematically analyze the transient but will investigate various means of minimizing it.

Several other arrangements of Fig. 2.18 can be devised to minimize the introduction of the initial conditions. For example, Fig. 2.19 shows an arrangement that is similar to Fig. 2.18 except that the switch does not select between the lead network and the short circuit channel but merely opens the short circuit.

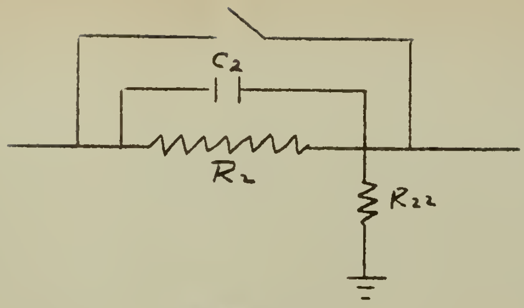


Fig. 2.19

With this arrangement no current will flow thru C_2, R_2 & R_{22} because the grid of the following amplifier is also assumed to be at ground potential and all the current will flow thru the short circuit branch. There will then be a greater difference of potential before and after switching because now the potential after switching is ground potential. Fig. 2.20 shows the response under these conditions and a comparison with Fig. 2.17 will show that this response is even more sluggish.

Another arrangement is shown in Fig. 2.21.

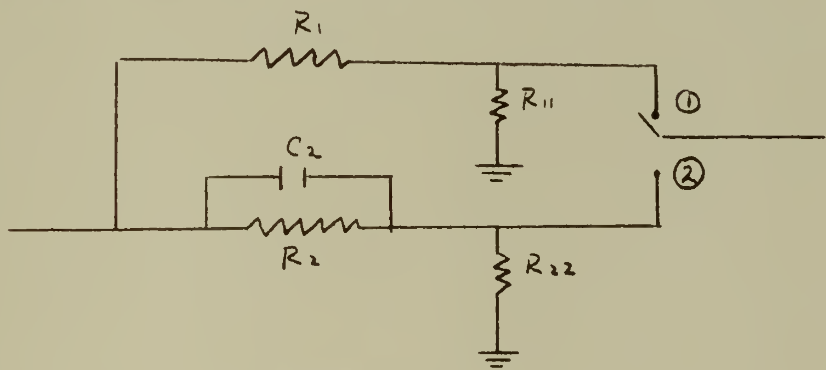


Fig. 2.21

This arrangement introduces a parallel set of resistors in the short circuit branch in an attempt to equalize the voltage at points ① and ② at the instant of switching. The reasoning here is that it may be possible to find a ratio of R_1/R_{11} and R_2/R_{22} for a given C_2 & $J_1 J_2$ that will produce equal voltages at points ① and ② at the instant of switching. The design problem in this case is quite complex because the damping ratios are de-

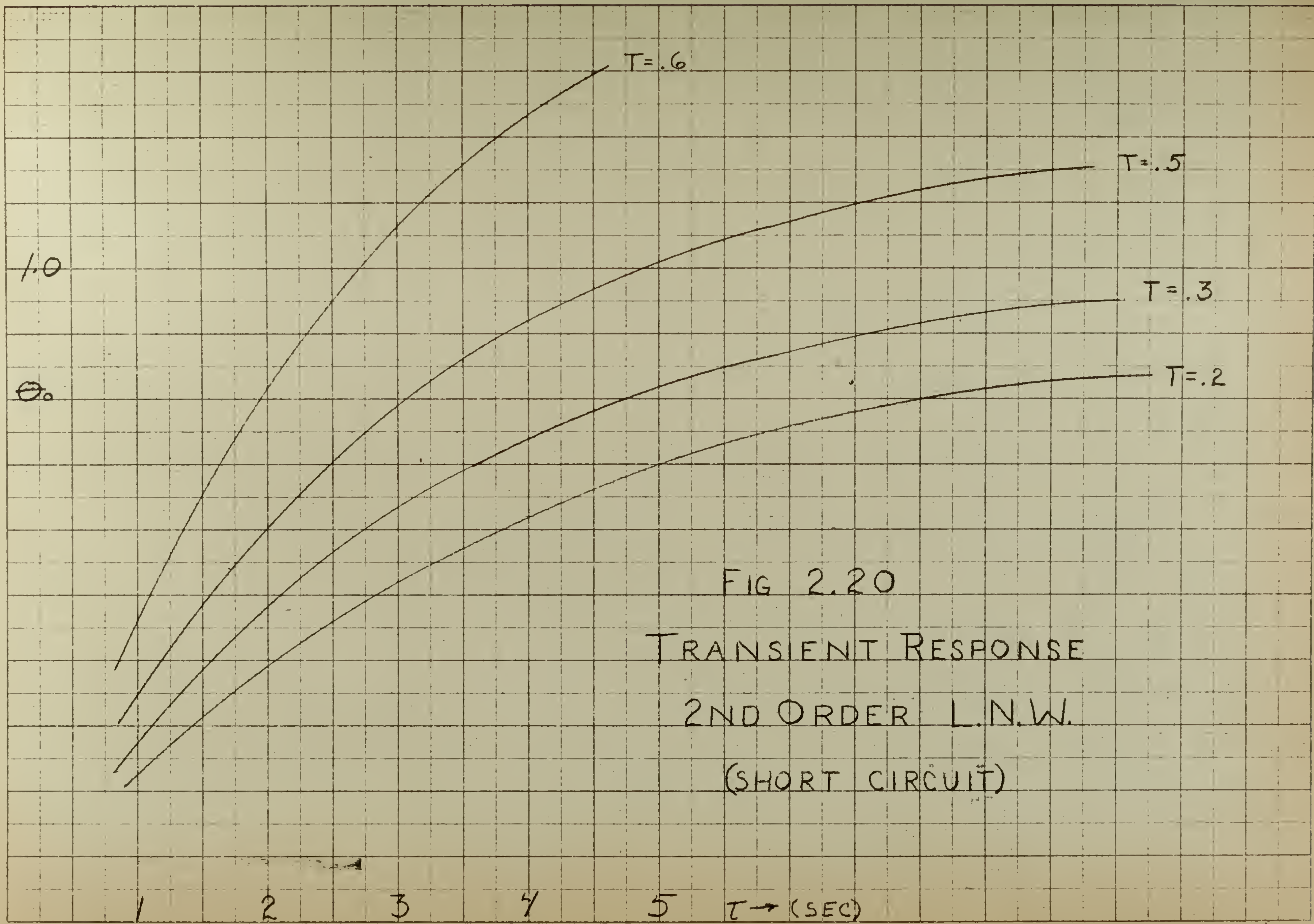


FIG 2.20
TRANSIENT RESPONSE
2ND ORDER L.N.W.
(SHORT CIRCUIT)

pendent on the valuesof the components in the lead network and parallel resistance network, the switching time is dependent on the damping ratios and therefore on the values of the components. Therefore, it may not be possible to find a ratio of resistances that will satisfy the condition of equal voltages between points 1 and 2 at the time of switching and also satisfy the conditions necessary to produce the desired damping ratios.

The parallel resistance network will be considered and analyzed in the third order system (Chapter III B).

III THIRD ORDER SYSTEMS

This Chapter will investigate the advantages and disadvantages of applying delayed compensation to third order systems. Both methods of compensation are again considered, namely, tachometer feedback and lead network compensation. Both methods are analyzed by employing the root locus method. A rudimentary background in the root locus method is assumed in this Chapter.

The same approach is taken here that was taken in the case of the second order systems. Namely, prior to the introduction of the compensation the system should be highly underdamped even to the point of approaching instability (a very low ζ_1). After the introduction of the compensation the system should be highly over damped in order to bring it into correspondence quickly (a high ζ_2).

A) TACHOMETER FEEDBACK COMPENSATION

1) MATHEMATICAL ANALYSIS

Condition I

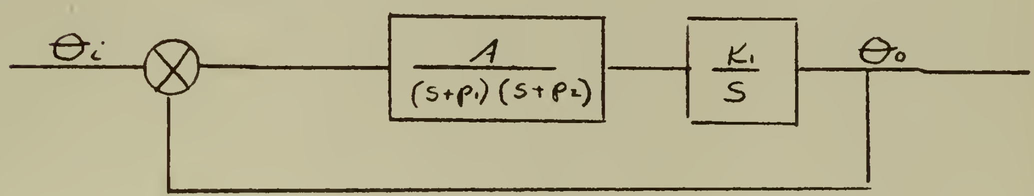


Fig. 3.1

$$G_{01} = \frac{AK_1}{s(s+p_1)(s+p_2)}$$

poles of $G_{01} = 0, p_1, p_2$

In order to achieve a very low ζ_1, the forward gain of the system is adjusted until the roots of G_{c1} lie in the general vicinity of the positive imaginary axis as shown in Fig. 3.2.

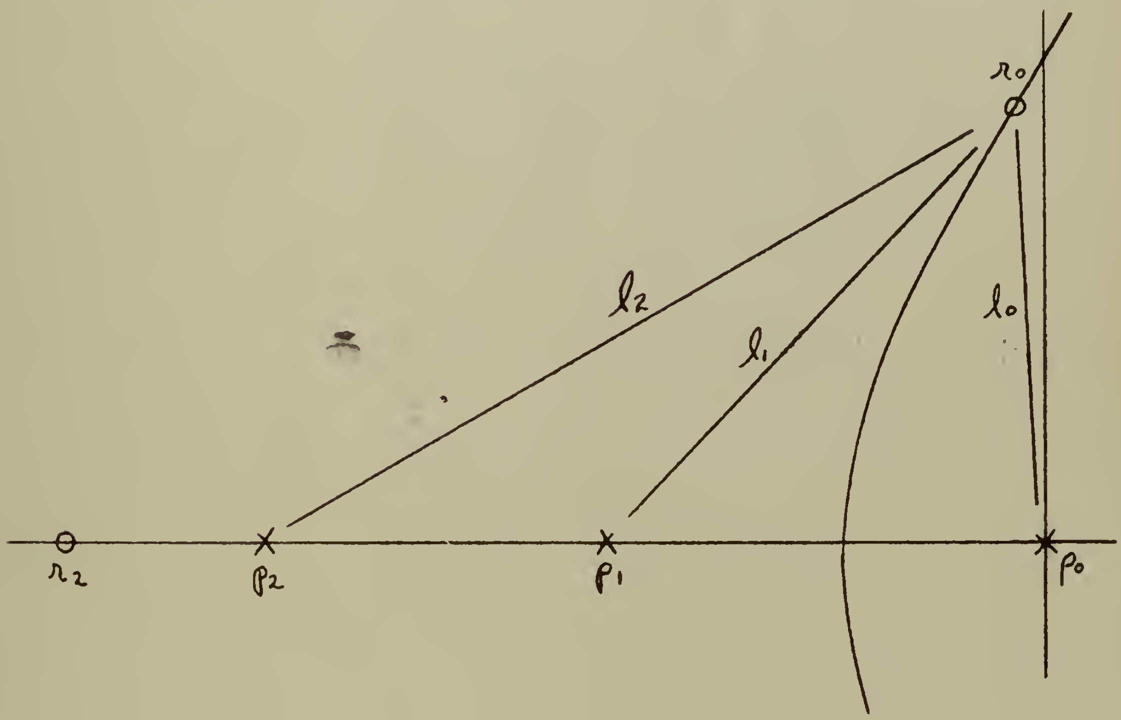


Fig. 3.2

The magnitude condition of the root locus theory demands that

$$l_1 l_2 l_0 = A K_1 \tag{3.1}$$

Condition II

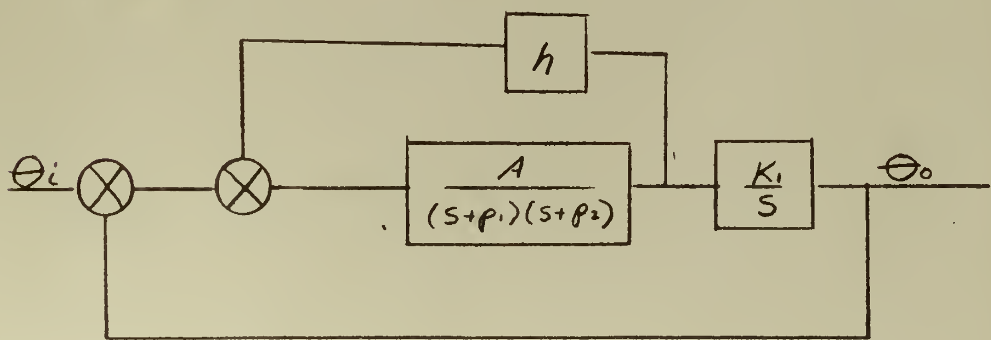


Fig. 3.3

$$G_{o2} = \frac{\dfrac{AK_1}{s(s+p_1)(s+p_2)}}{1 + \dfrac{Ah}{(s+p_1)(s+p_2)}} = \frac{AK_1}{s(s+p_1)(s+p_2) + Ahs}$$

$$= \frac{AK_1}{s\left[s^2 + (p_1+p_2)s + (p_1p_2 + Ah)\right]}$$

The poles of G_{o2} now are zero and the roots of the above quadratic

3.2) $P_0 = 0$

3.3) $P_1 = -\frac{p_1+p_2}{2} + \frac{1}{2}\sqrt{(p_1-p_2)^2 - 4Ah}$

3.4) $P_2 = -\frac{p_1+p_2}{2} - \frac{1}{2}\sqrt{(p_1-p_2)^2 - 4Ah}$

In order for ζ_2 to be large (overdamped) the roots of G_{c2} must lie on the negative real axis. This necessarily requires that the poles of G_{o2} also lie on the negative real axis. If the poles of G_{o2} are imaginary then the roots of G_{c2} can never be real in a third order system of this sort. Fig. 3.4 graphically represents the reasoning here.

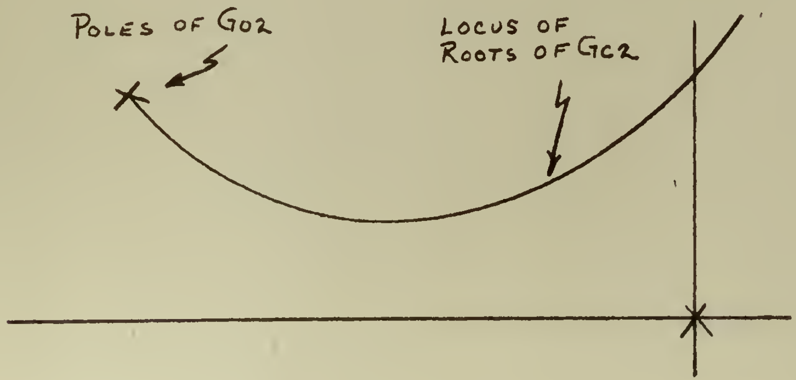


Fig. 3.4

Therefore it can be reasoned from equations 3.3 and 3.4 that

3.5) $(\rho_1 - \rho_2)^2 \geq 4Ah$

This imposes a critical restriction on the range of possible values for h. If the K_1 term were applied before the feedback loop in Fig. 3.3 the resulting restriction on h would be $(\rho_1 - \rho_2)^2 \geq 4AhK_1$ which is an even stricter restriction on the range of possible values for h.

Fig. 3.5 represents the general configuration of the root locus for Condition II.

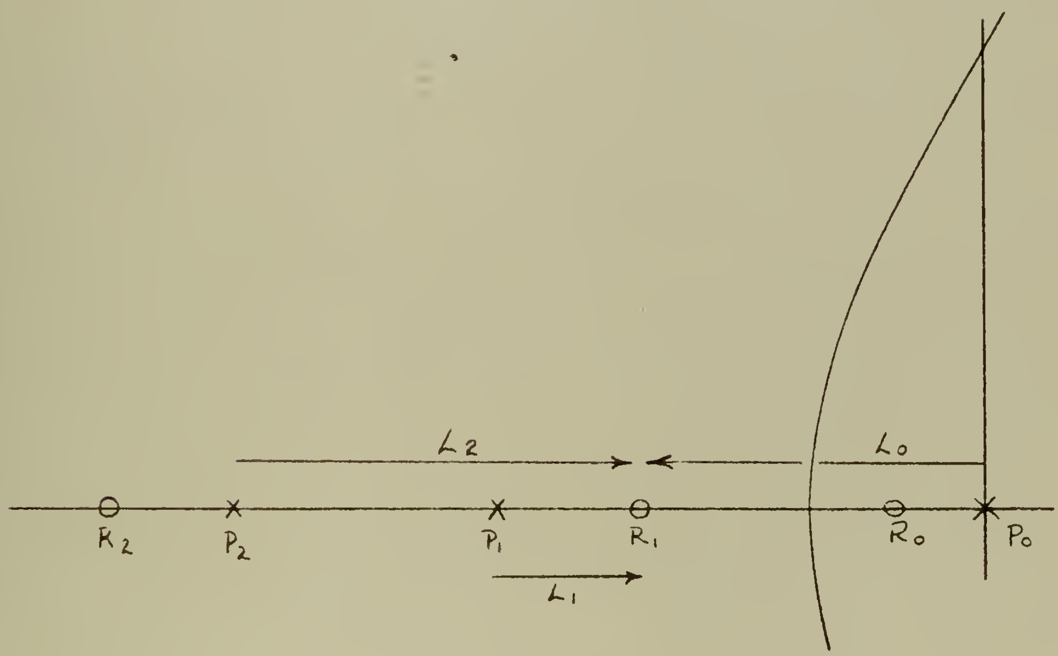


Fig. 3.5

The gain for condition II must be low enough such that R_0 + R_1 lie on the negative real axis (high J_2). The magnitude condition again demands that 3.6) $L_0 L_1 L_2 = AK_1$

It is now desirable to determine the behavior of the location of P_1 + P_2 as h is varied through its narrow range of zero to $(\rho_1 - \rho_2)^2/4A$.

Consider now only the portion of Fig. 3.3 that is shown in Fig. 3.6

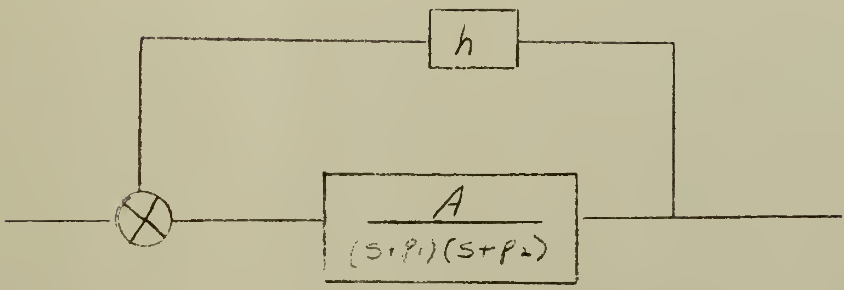


Fig. 3.6

The above figure cannot be realized physically except in a computer analysis but permits a clearer concept of pole variations and therefore is used in this discussion.

$$G_c = \frac{\dfrac{A}{(s+p_1)(s+p_2)}}{1+\dfrac{Ah}{(s+p_1)(s+p_2)}} = \frac{A}{(s+p_1)(s+p_2)+Ah}$$

$$G_c = \frac{A}{s^2+(p_1+p_2)s+(p_1p_2+Ah)}$$

a second order system whose poles are:

$$-\tfrac{1}{2}(p_1+p_2) \pm \tfrac{1}{2}\sqrt{(p_1+p_2)^2-4(p_1p_2+Ah)}$$

OR

$$-\tfrac{1}{2}(p_1+p_2) \pm \tfrac{1}{2}\sqrt{(p_1-p_2)^2-4Ah}$$

the root locus for this condition is

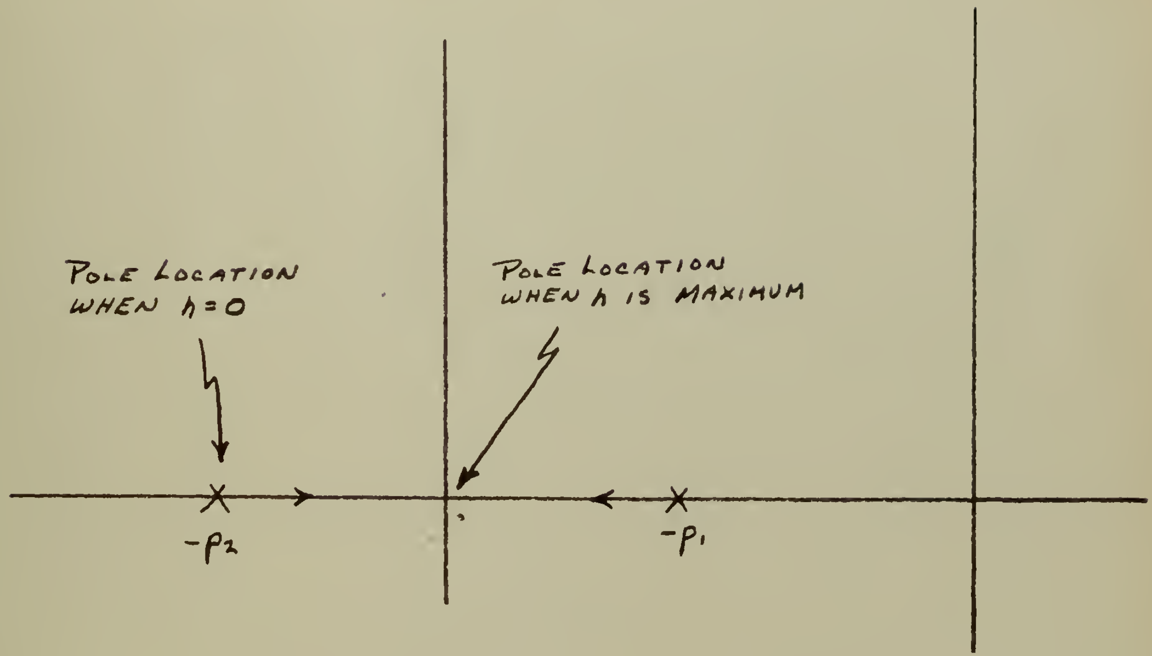


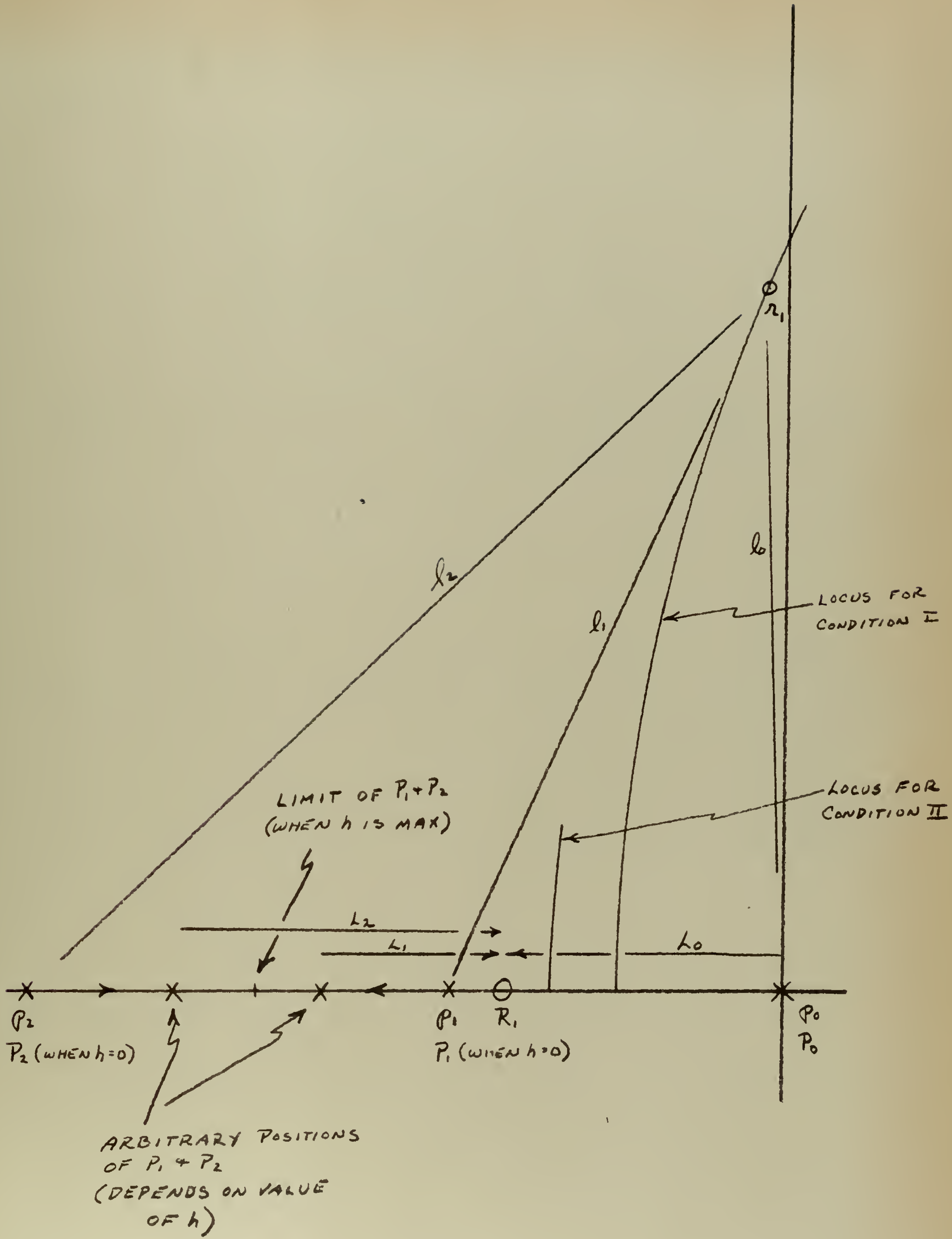
Fig. 3.7

When h is at its lower limit (equal to zero) the roots of G_0 are $-p_1, -p_2$. When h is at its upper limit $((p_1-p_2)^2/4A$) the roots of G_0 are identical and lie on the real axis at the intersection of the root locus.

The preceding can now be summarized graphically in Fig. 3.8 which is merely Fig. 3.5 superimposed on Fig. 3.2.

A study of Fig. 3.8 shows that it is impossible (except in very special cases) to satisfy both equations 3.1 and 3.6 with the same value of AK_1. This is true regardless of the chosen value of h or the actual and/or relative sizes of the roots of the third order system.

This problem can be circumvented by having available two different values of AK_1. One for Condition I and one for Condition II. This can be achieved in a variety of ways. For instance, the value of K_1 can be



r_1
l_2
l_1
l_0
LOCUS FOR CONDITION I
LOCUS FOR CONDITION II
LIMIT OF P_1 + P_2 (WHEN h IS MAX)
L_2
L_1
L_0
p_2
P_2 (WHEN h=0)
p_1
R_1
P_1 (WHEN h=0)
p_0
P_0
ARBITRARY POSITIONS OF P_1 + P_2 (DEPENDS ON VALUE OF h)

FIG 3.8

reduced at the instant the tach channel switch closes by the following method.

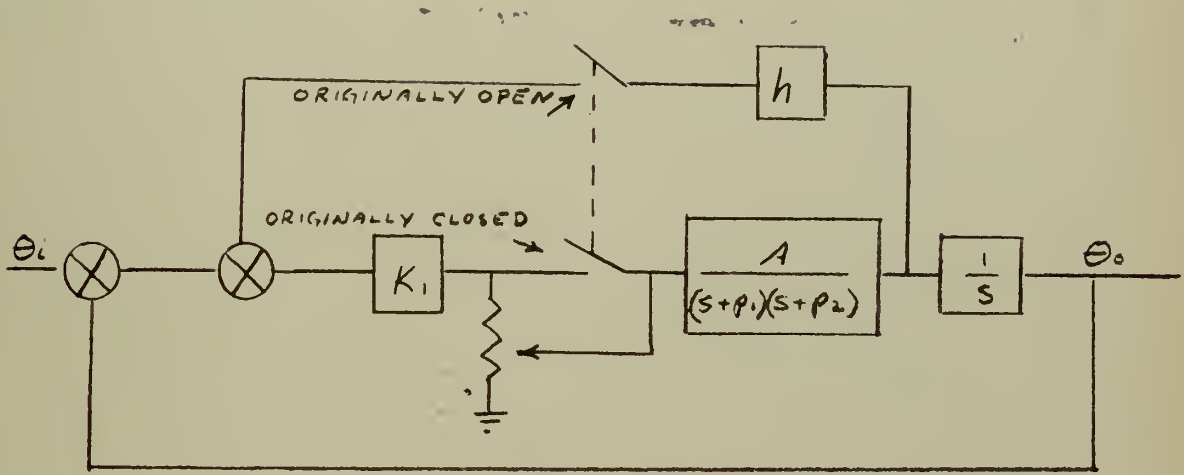


Fig. 3.9

The arrangement in Fig. 3.9 along with other possible schemes is theoretically possible to construct and operate. The increased complexity of the hardware, however, suggests investigation of other means of compensation for third order systems. For this reason, the investigation of tachometer feedback compensation for third order systems is not carried any farther in this paper.

B) LEAD NETWORK COMPENSATION

1) MATHEMATICAL ANALYSIS

Delayed compensation is achieved with a lead network by the arrangement in Fig. 3.10

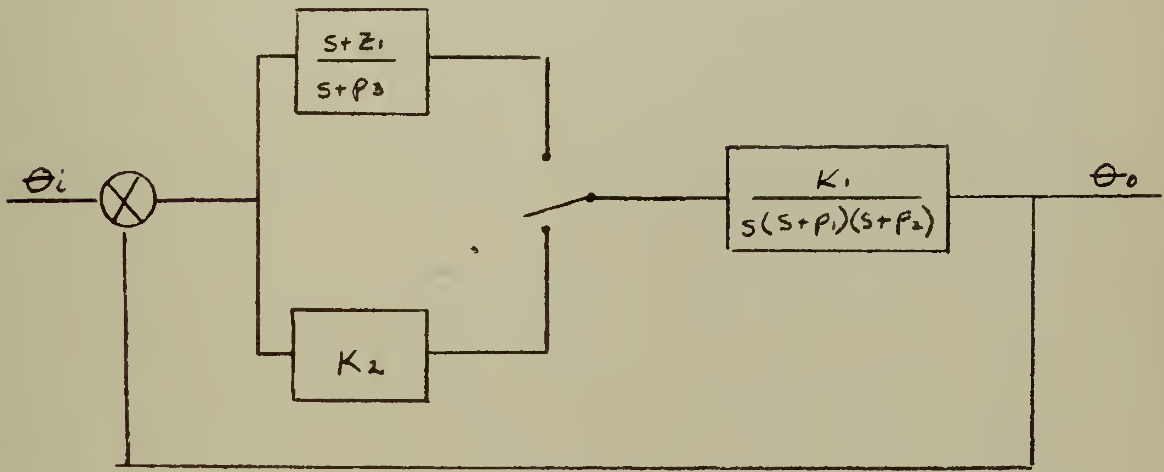


Fig. 3.10

The actual circuitry of the lead network is shown in Fig. 3.11

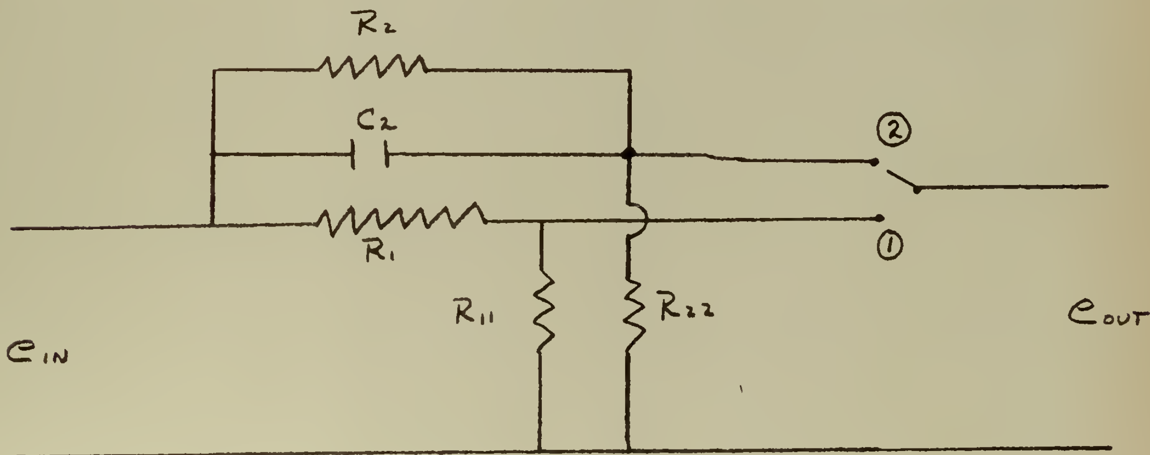


Fig. 3.11

The lead network consists of R_2 C_2 & R_{22} and will be in the circuit when the switch is in position ② . R_1 and R_{11} parallel the lead net-

work and are in the circuit when the switch is in position ① . The parallel circuit is introduced here in order to make the potential at points ① and ② as nearly equal as possible at the instant of switching. If this is accomplished then the transient effect due to switching will be minimized. Because the parallel circuit acts as an attenuator the resulting K_2 will be less than unity. In the following discussion, however, it will be assumed that K_2 has been compensated with an amplifier and the resulting K_2 is equal to unity. This allows the analysis to be based on the much simpler arrangement of Fig. 3.12.

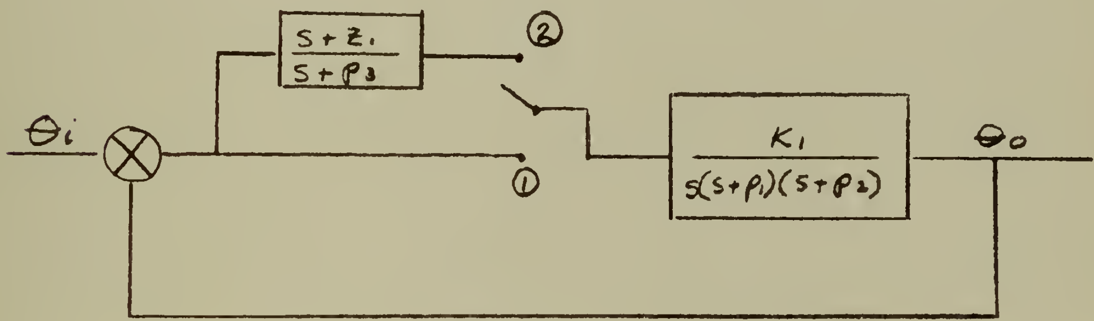


Fig. 3.12

During the first portion of the response when a low ζ is desired the switch is in position ① . As the response nears correspondence the switch is thrown to position ② thereby introducing the lead network and increasing the damping ratio.

Condition I

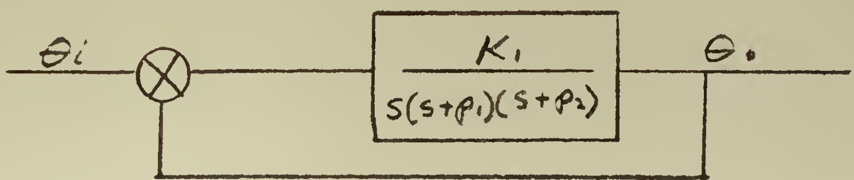


Fig. 3.13

The root locus for the system during condition I is depicted below.

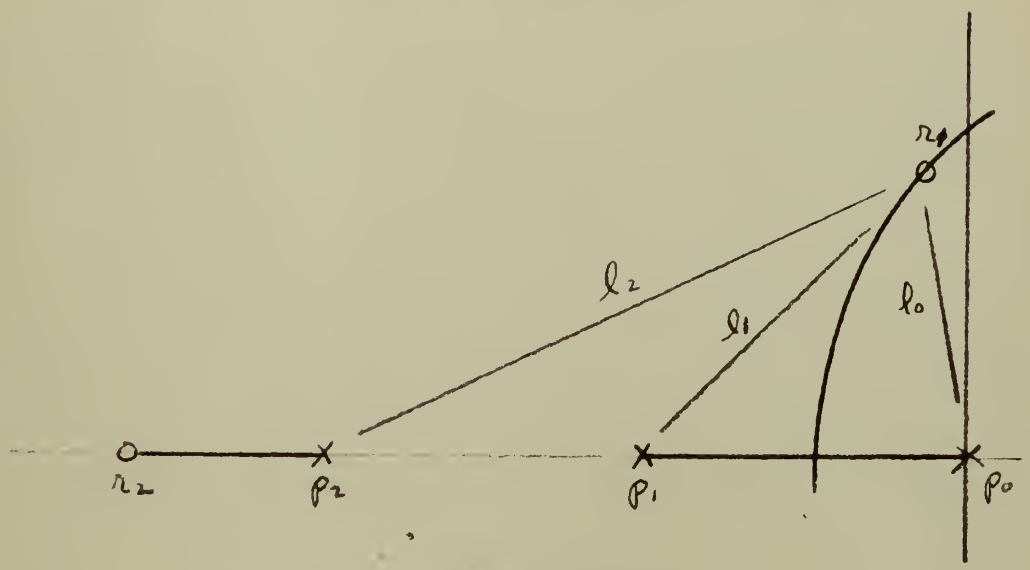


Fig. 3.14

K_1 is adjusted so that the complex roots are close to the imaginary axis thus producing a highly oscillatory system (low ζ_1).

Condition II

The zero in the lead network is designed to cancel out one of the poles of the system.

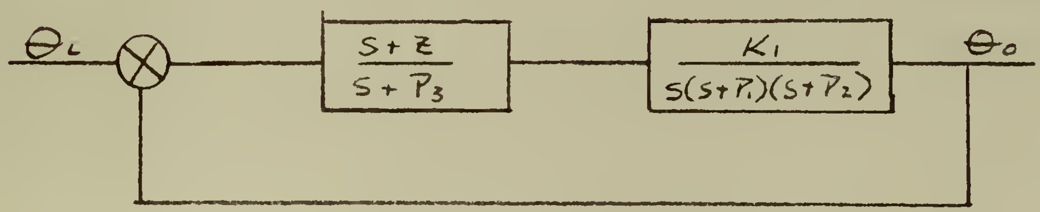


Fig. 3.15

In Fig. 3.15 $Z = P_1 = p_1$ AND $P_2 = p_2$. The root locus of Fig. 3.15 is shown below.

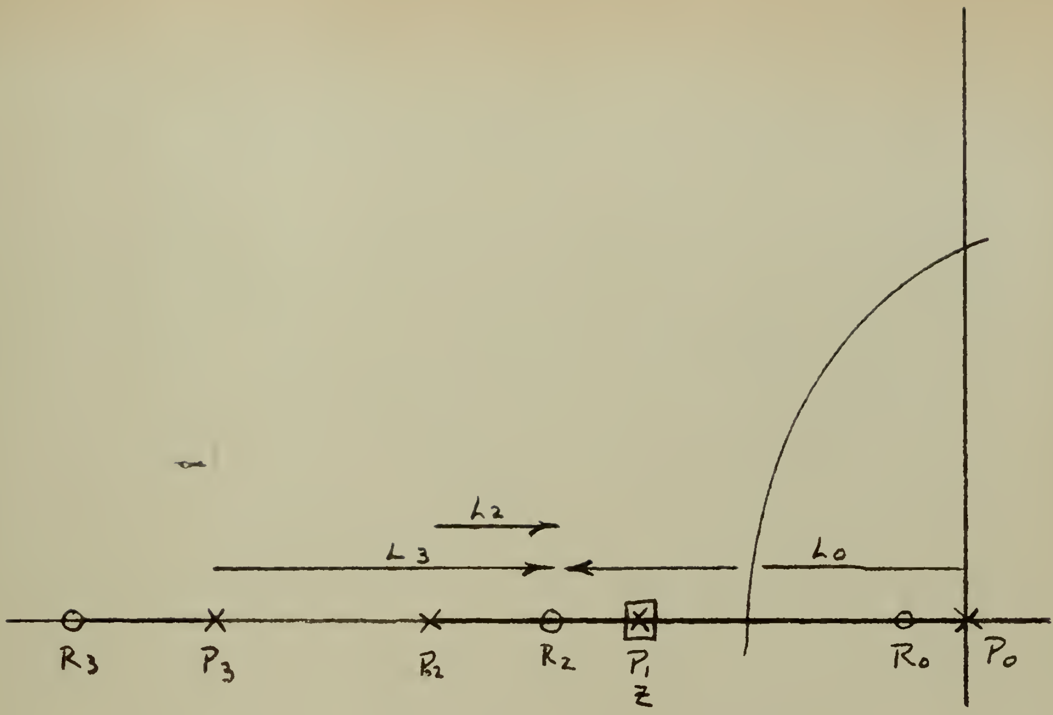


Fig. 3.16

P_3 is so positioned that it yields a high ζ_2 by producing roots of G_c on the real axis. The position of the roots on the axis determines the magnitude of ζ_2. It must be kept in mind, however, that the magnitude condition presents the same problem here as it did with tachometer feedback compensation. In order to have the same amplification produce both the low ζ in condition I and the high ζ in condition II the following must be true:

$$l_0 l_1 l_2 = K_1 = L_0 L_2 L_3$$

The procedure for determining the location of P_3 and at the same time produce the desired damping ratios is presented in the design section. (Chapter III B3)

2) COMPUTER STUDY

The block diagram of the computer set up is shown in Fig. 3.17.

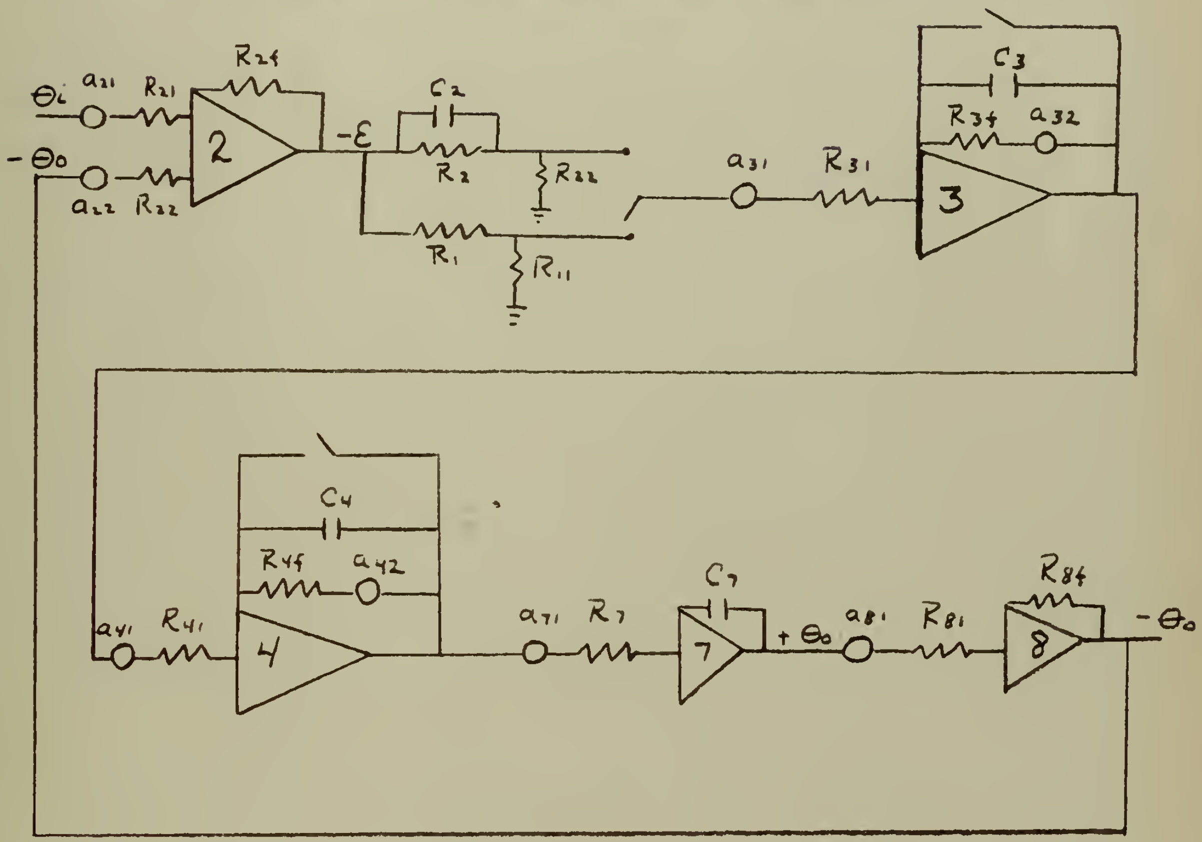


Fig. 3.17

Fig. 3.18 shows a family of curves with $\zeta_1 = .033$, $\zeta_2 = 2$ and T varied from .256 seconds to .766 seconds.

Here again the system is somewhat sluggish due to the initial conditions introduced at the time of switching. However, the responses in Fig. 3.18 are an improvement over the second order lead network responses where the parallel resistance network was not employed. The values of the resistors in the parallel network were chosen intuitively and apparently did not exactly equalize the voltages at points (1) & (2) at the time

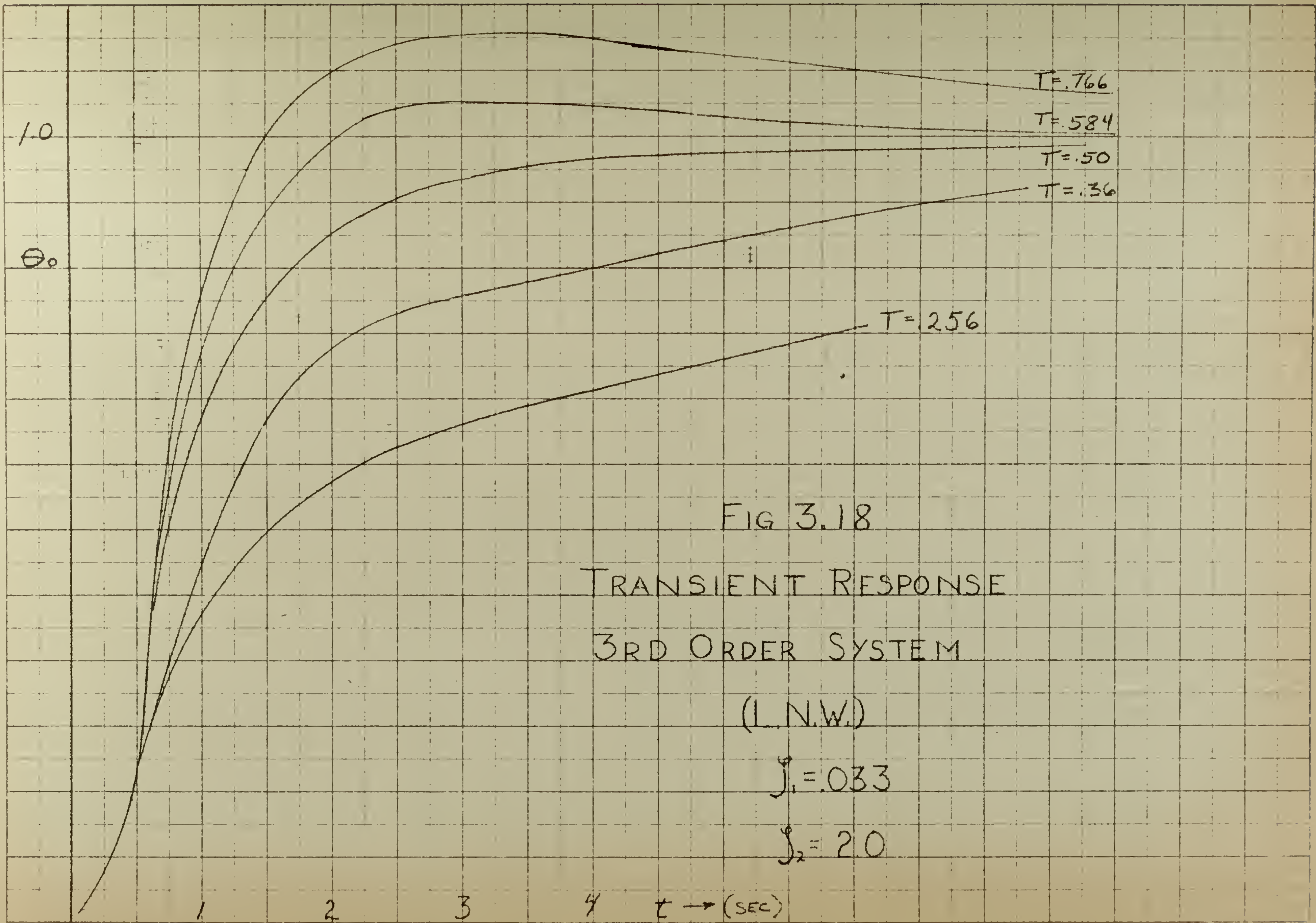


FIG 3.18
TRANSIENT RESPONSE
3RD ORDER SYSTEM
(L.N.W.)
$\zeta_1 = .033$
$\zeta_2 = 2.0$

of switching. It did, however, reduce the effects of the introduced initial conditions.

3) DESIGN CONSIDERATIONS

As in the section on design of second order systems, this section will investigate the design problems that will be encountered in adapting delayed compensation to existing systems.

a) DESIGN CRITERIA

The design criteria for the third order system is not similar to that of the third order system. In the second order system the required torque was used as a basis for design, whereas in the third order system the pole-zero configuration is employed.

As pointed out in Chapter III B1 the pole of G_{o2} must be so positioned that:

-Condition II will produce an overdamped system

- K_1 will be the same for Condition I and Condition II

-The desired ζ_1 + ζ_2 can be obtained

In order to crystallize the solution to the design problem a numerical example will be analyzed. Assume an existing system as shown in Fig. 3.19. The design problem now is to add a lead network and an amplifier to the system and thereby improve the response by employing the delayed damping method.

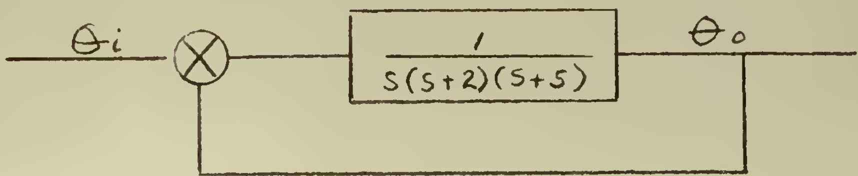


Fig. 3.19

b) CONDITION I

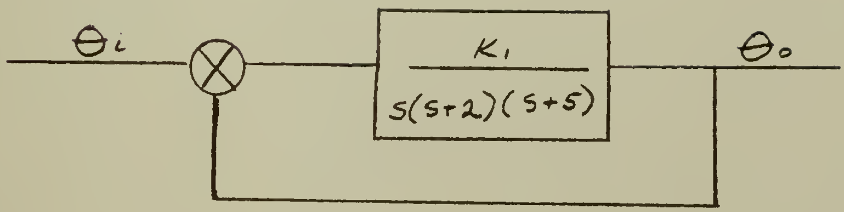


Fig. 3.20

$$G_{c1} = \frac{K_1}{s(s^2+7s+10)+K_1}$$

K_1 is determined graphically in Fig. 3.21 by placing the roots near the imaginary axis on the root locus. This will yield a low ζ_1

$$K_1 = l_0 \, l_1 \, l_2$$

$$= (2.98)(3.5)(5.75) = 60$$

3.7) $$G_{c1} = \frac{60}{s^3+7s^2+10s+60} = \frac{r_2 \omega_n^2}{(s+r_2)(s^2+2\zeta\omega_n s+\omega_n^2)}$$

To determine ζ_1

Find the real root of eq'n 3.7 by:

a) numerical methods

b) dividing through by $r_1 + r_0$ (arbitrarily placed on the root locus)

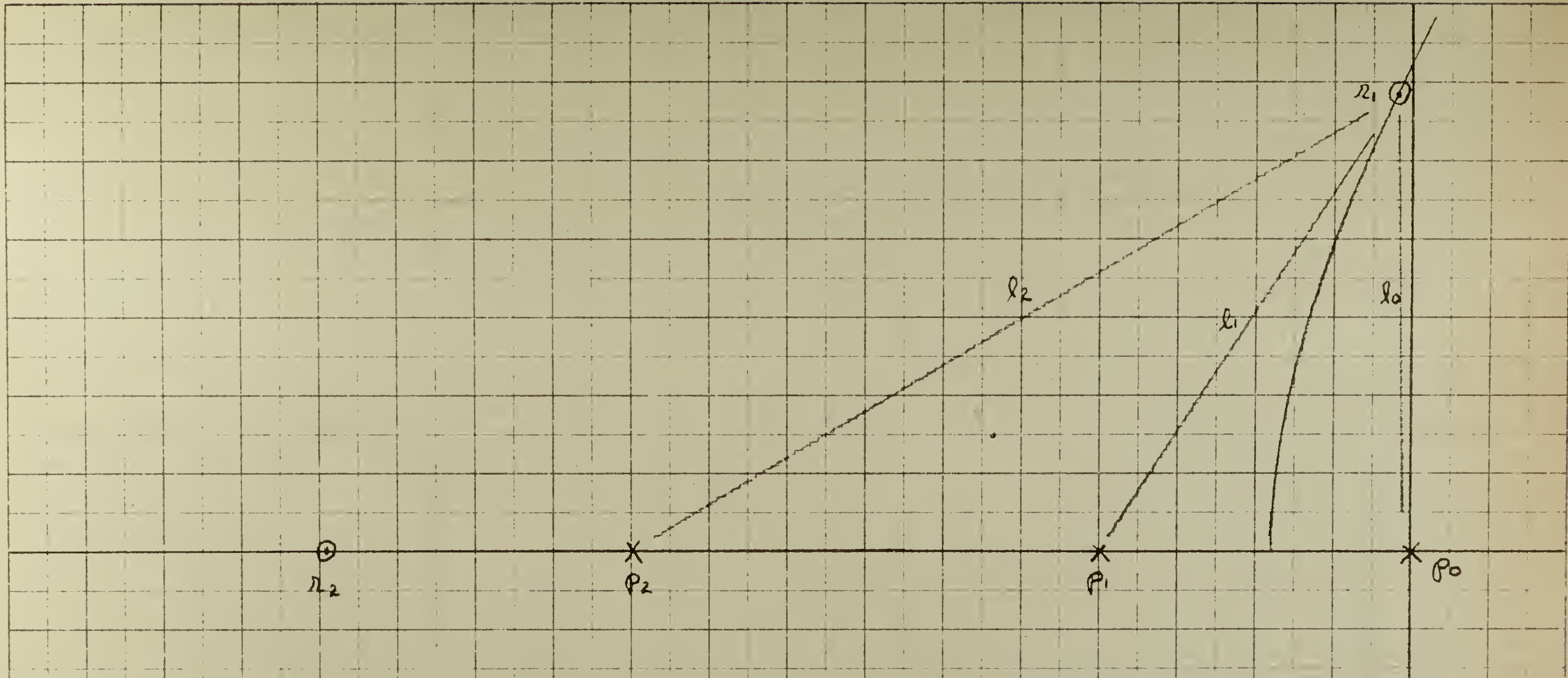


FIG 3.21

ROOT LOCUS PLOT

CONDITION II

$$r_2 = -6.987$$

$$\left.\begin{aligned} r_1 &= -.1 + 3i \\ r_0 &= -.1 - 3i \end{aligned}\right\} \text{ CHOSEN POSITION NEAR IMAGINARY AXIS}$$

$$2\zeta\omega_n = r_1 + r_0 = .2$$

$$\omega_n^2 = r_1 r_0 = .01 + 9 = 9.01$$

$$\omega_n \doteq 3$$

3.8) $$\zeta_1 = \frac{.2}{2(3)} = .0333$$

c) **CONDITION II**

Now the problem is met of placing P_3 such that the three conditions are satisfied. In this example a desired ζ_2 of 2 will be a design requirement.

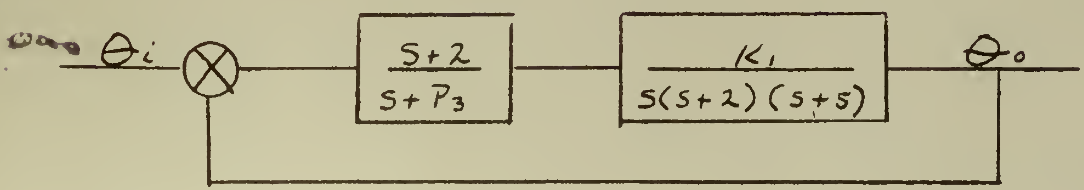


Fig. 3.22

$$3.9)\quad G_{c2} = \frac{60}{s(s+5)(s+P_3)+60} = \frac{R_3\,\omega_n^2}{(s+R_3)(s+2\zeta\omega_n+\omega_n^2)}$$

The location of P_3 for a desired ζ equal to 2 can be accomplished in two ways.

FIRST- ANALYTICAL

The roots of the denominator of eq'n 3.9 can be determined in terms of P_3 (numerical method). Then the following conditions must hold:

a) the sum of the two right hand roots must equal $2\zeta\omega_n$ or $4\omega_n$

b) the left hand root times ω_n^2 must equal 60.

These two conditions will produce two equations and two unknowns

$$f(P_3) = 4\omega_n$$

$$F(P_3)\,\omega_n^2 = 60$$

from these P_3 can be determined.

The analytical method will obviously require considerable time.

SECOND- GRAPHICAL (see Fig. 3.23)

The location of P_3 can be determined much quicker by graphical iteration and for that reason will be employed here. The same conditions must be met here as in the analytical method. Represented in graphical notation they are:

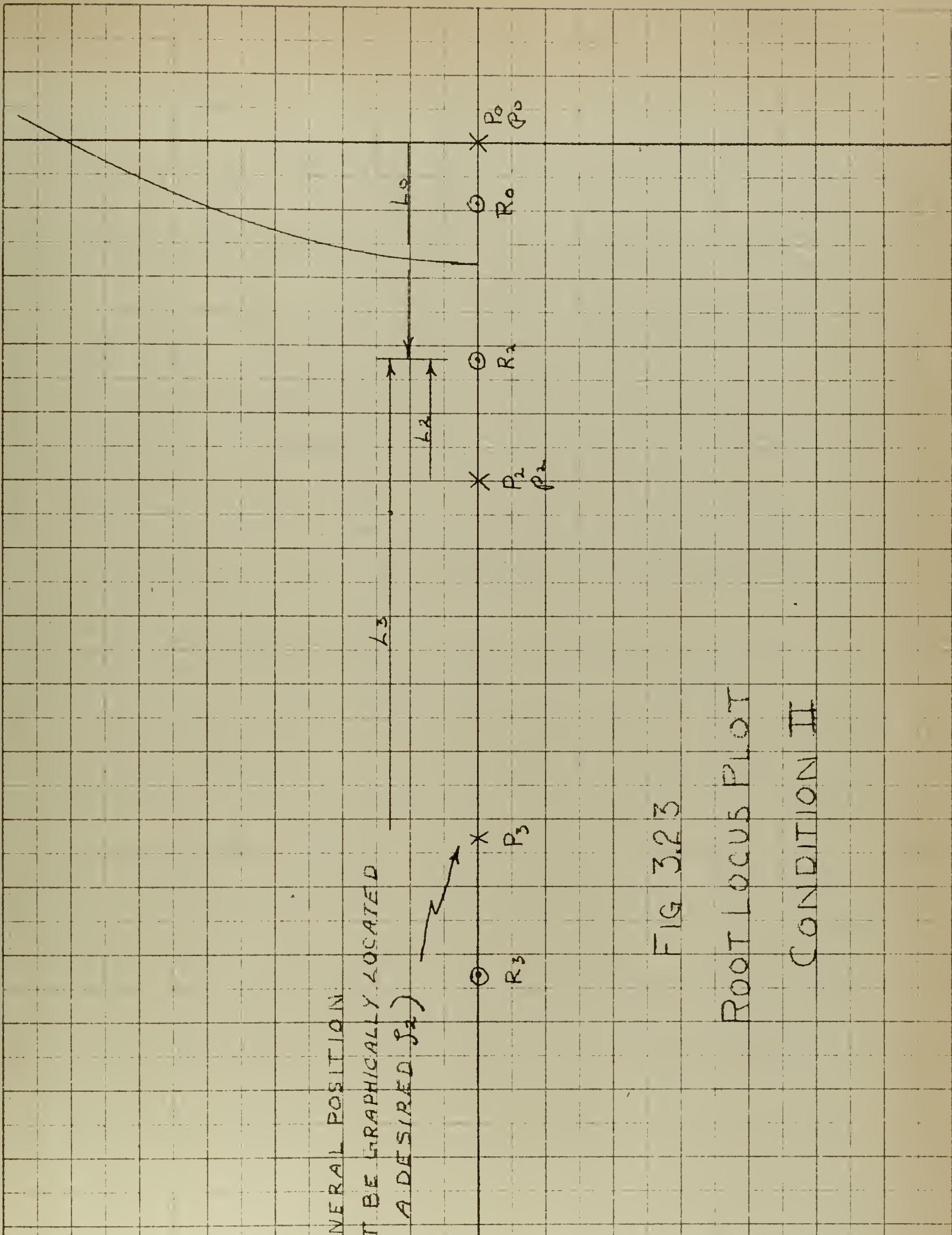


FIG 3.23

ROOT LOCUS PLOT

CONDITION II

3.10) $L_3 L_2 L_0 = 60$

3.11) $\frac{R_0 + R_2}{2\omega_m} = \zeta$ (= 2 IN THIS CASE)

3.12) $\omega_m = \sqrt{\frac{60}{R_3}}$

This iteration involves two variable points, P_3 and any one of the three roots of G_{C2} (R_0 R_1 OR R_3)

OUTLINE OF GRAPHICAL METHOD (see Fig. 3.24)

a) Arbitrarily choose a position for R_2 between the breakaway point and P_2

b) Determine the location of P_3 by eq'n 7.11 $\left(P_3 = \frac{60}{L_0 L_2}\right)$

c) Choose a position for R_3 and adjust it until $L_0 L_2 L_3 = 60$ (the L's in this case terminate at R_3)

d) Locate R_0 by the same method as R_3 was located

e) Now determine ζ_2 by eq'ns 3.11 and 3.12

f) If the resulting ζ is not the desired ζ return to step a), adjust R_2, and repeat the five steps. Continue this until the desired ζ_2 is achieved.

When the location of P_3 is adjusted the position of the breakaway point is also altered. This is ignored in Fig. ~~24~~ 3.24 so as ~~in~~ not to confuse the picture. This change in position of the breakaway point in no way affects the results of the graphical method be ause during Condition II all the roots are necessarily on the real axis and uually very close to their respective poles.

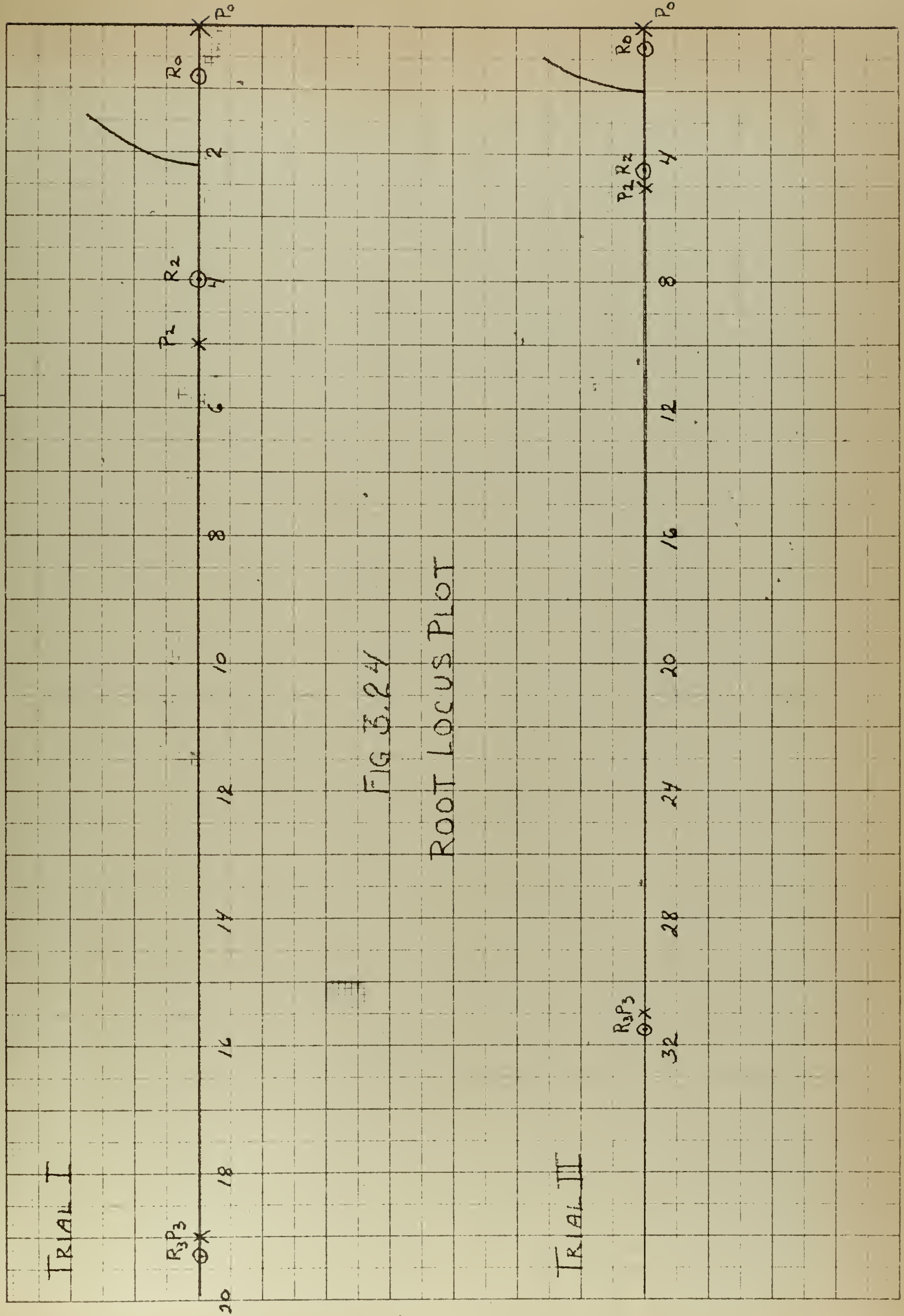


FIG 3.24
ROOT LOCUS PLOT

TRIAL I

a) R_2 POSITIONED AT -4

b) $L_3 = \frac{60}{(1)(4)} = 15$

P_3 NOW LOCATED AT $-15-4=-19$

c) (R_3 AT -20) $(20)(1)(15) = 300$ TOO HIGH

(R_3 AT -19.5) $(19.5)(.5)(14.5) = 141$ TOO HIGH

(R_3 AT -19.3) $(19.3)(.3)(14.3) = 82.6$ TOO HIGH

(R_3 AT -19.22) $(19.22)(.22)(14.22) = 60.4$

d) NOW POSITION R_0

(R_0 AT 1) $(1)(4)(18) = 72$ TOO HIGH

(R_0 AT .9) $(.9)(4.1)(18.1) = 67$ TOO HIGH

(R_0 AT .8) $(.8)(4.2)(18.2) = 61$

e) NOW DETERMINE ζ_2

$\omega_n = \sqrt{60/19.22} = 1.77$ (EQ'N 3.12)

$R_0 + R_2 = 4 + .8 = 4.8$

$\zeta_2 = \frac{4.8}{2(1.77)} = 1.36$ (EQ'N 3.11)

THIS IS LOWER THAN THE DESIRED ζ_2. REPEAT THE ABOVE PROCEDURE AFTER MOVING R_2 CLOSER TO P_2.

TRIAL II

a) R_2 AT 4.5

b) $$L_3 = \frac{60}{(.5)(4.5)} = 26.6$$

P_3 NOW LOCATED AT $-26.6 - 4.5 = -31.1$

c) (R_3 AT -31.2) $(.1)(31.2)(26.2) = 81.5$ TOO HIGH

(R_3 AT -31.175) $(.075)(30.175)(26.175) = 59.3$

d) (R_0 AT .6) $(.6)(3.9)(30.5) = 71.5$ TOO HIGH

(R_0 AT .5) $(.5)(4)(30.6) = 61.2$

e) $$\omega_n = \sqrt{60/31.175} = 1.39$$

$$\zeta_2 = \frac{4.5 + .5}{2(1.39)} = \frac{5}{2.78} = 1.8$$

CLOSE ENOUGH TO THE DESIRED VALUE OF 2

The properties of the lead network and the necessary amplification are now known. The complete system is shown in Fig. 3.25.

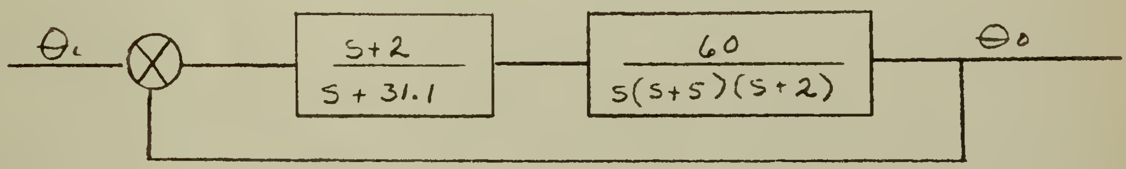


Fig. 3.25

The lead network components are calculated as follows (see Chapter II B 1)

$$\frac{1}{R_2 C_2} = 2 \qquad \text{LET} \quad C_2 = 1.0\ \mu f$$
$$R_2 = .5\ M\Omega$$

$$\frac{R_{22} + R_2}{R_{22} R_2 C_2} = 31.1$$

$$\frac{R_{22} + .5}{.5 R_{22}} = 31.1$$

$$R_{22} + .5 = 15.55 R_{22}$$

$$R_{22} = \frac{.5}{14.55} = .0343\ M\Omega$$

IV PRACTICAL CONSIDERATIONS

Chapter I pointed out that the purpose of a servo system is to accurately duplicate any signal applied to it. The preceding chapters have pointed out methods whereby this accurate duplication may be approached.

If a system designed to yield a deadbeat response, such as that depicted in Fig. 2.5 were actually employed in the control of large bodies, two practical problems would immediately become obvious. First, at the instant of switching the system would have to withstand tremendous accelerations. This would eventually lead to mechanical stress failures in the mechanism. Second, again referring to Fig. 2.5, a small error in the delay time of .2 seconds results in an extremely long time to correspondence. In order to apply this method of control to a practical system these two problems must be solved.

Both the acceleration and accuracy of the time delay problems can be overcome simply by decreasing the value of ζ_2 from a very large value to a lower value. This lower value must necessarily still be higher than unity however.

The decreased ζ_2 will increase the radius of the path as it approaches correspondence. This naturally has the disadvantage of increasing the time to correspondence very slightly. Fig. 4.1 shows a family of transient responses with a $\zeta_1 = .165$ AND $\zeta_2 = 2.0$. Comparison of the deadbeat curves on Fig. 5.1 and Fig. 2.5 shows that the time to correspondence is practically the same in both. The increased radius of the path on Fig. 5.1 does, however, reduce the acceleration of the system considerably.

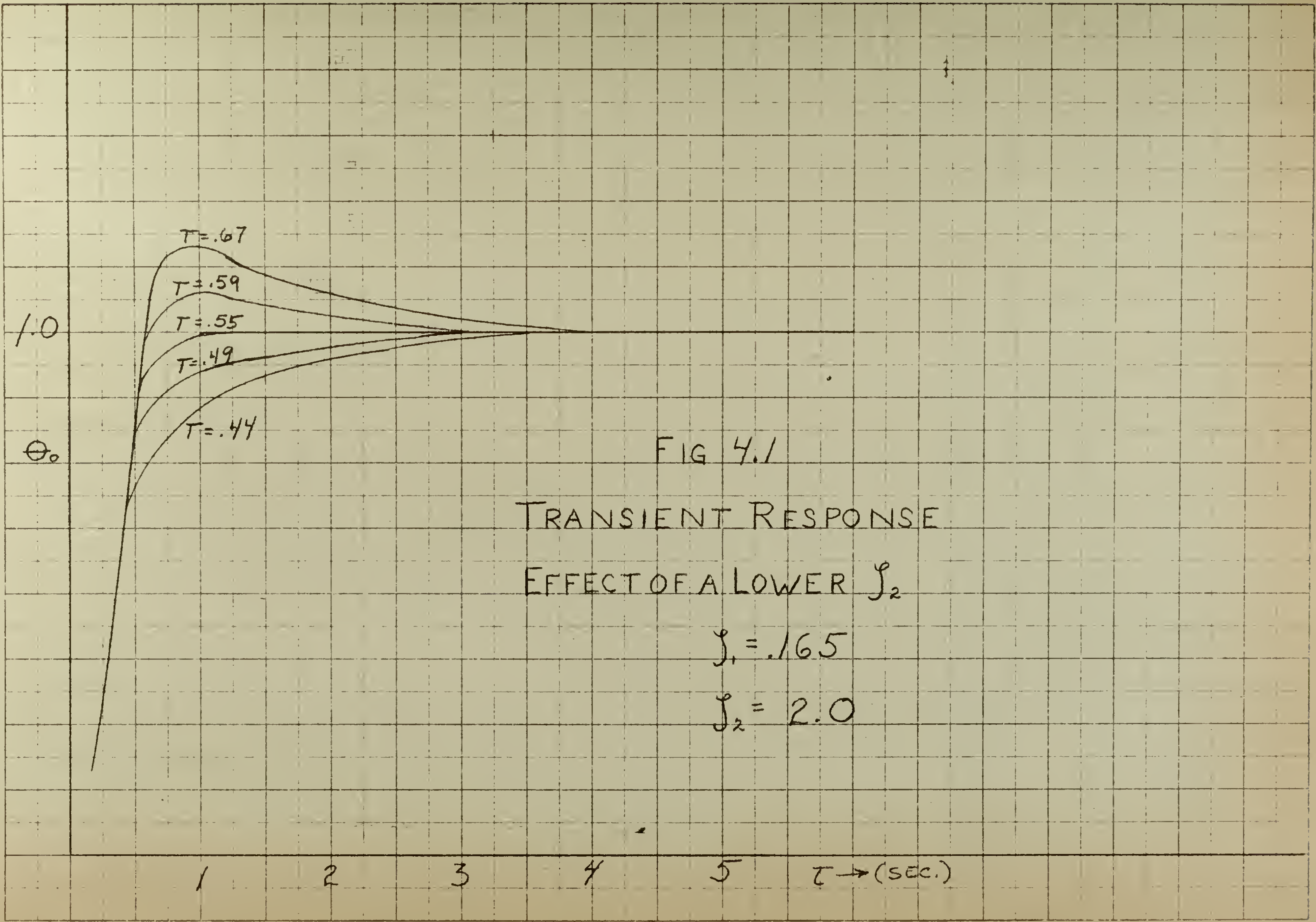


FIG 4.1

TRANSIENT RESPONSE

EFFECT OF A LOWER ζ_2

$\zeta_1 = .165$

$\zeta_2 = 2.0$

An error in the time delay of Fig. 5.1 does not result in such a long time to correspondence as it did with the system of the higher in Fig. 2.5.

When designing a system, therefore, the choice of the value of will have to be tempered with these practical considerations. If the aforementioned problems do not exist in a particular application, then $\mathcal{J}_2$ should be as high as possible (approach infinity) in order to obtain the optimum response.

V CONTROL OF TIME DELAY

A) AUTOMATIC CONTROL

It has been assumed in the previous chapters that the time delay has been controlled by some outside agency. This Chapter will consider a practical method for achieving this control.

If the values of $\dot{\theta}_o$ and $\mathcal{E}$ are plotted on a time axis they will have the form indicated in Fig. 5.1.

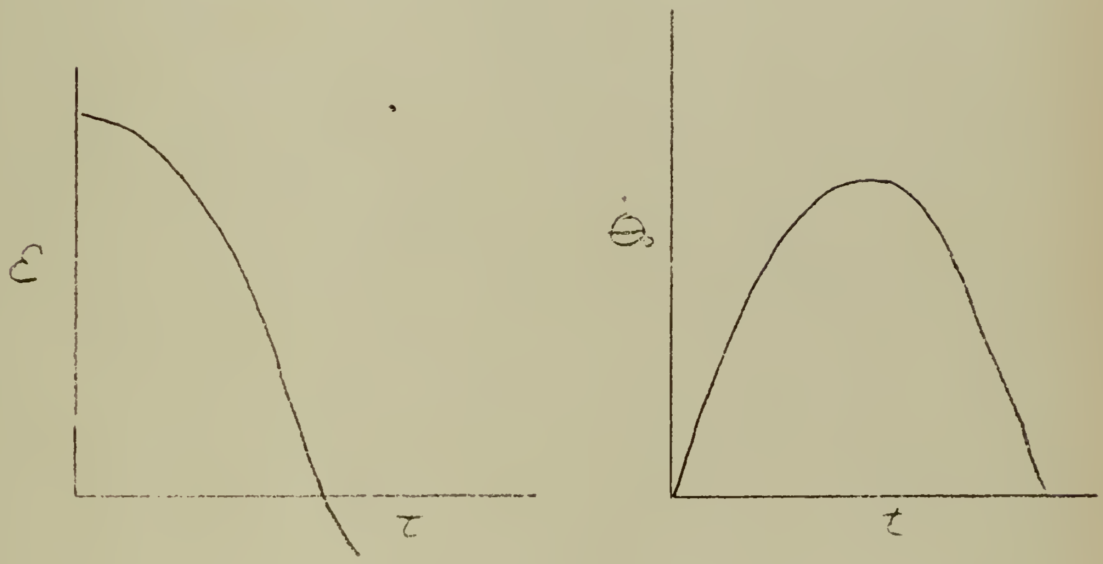


Fig. 5.1

Placing both on the same set of coordinates their sum will be as shown in Fig. 5.2.

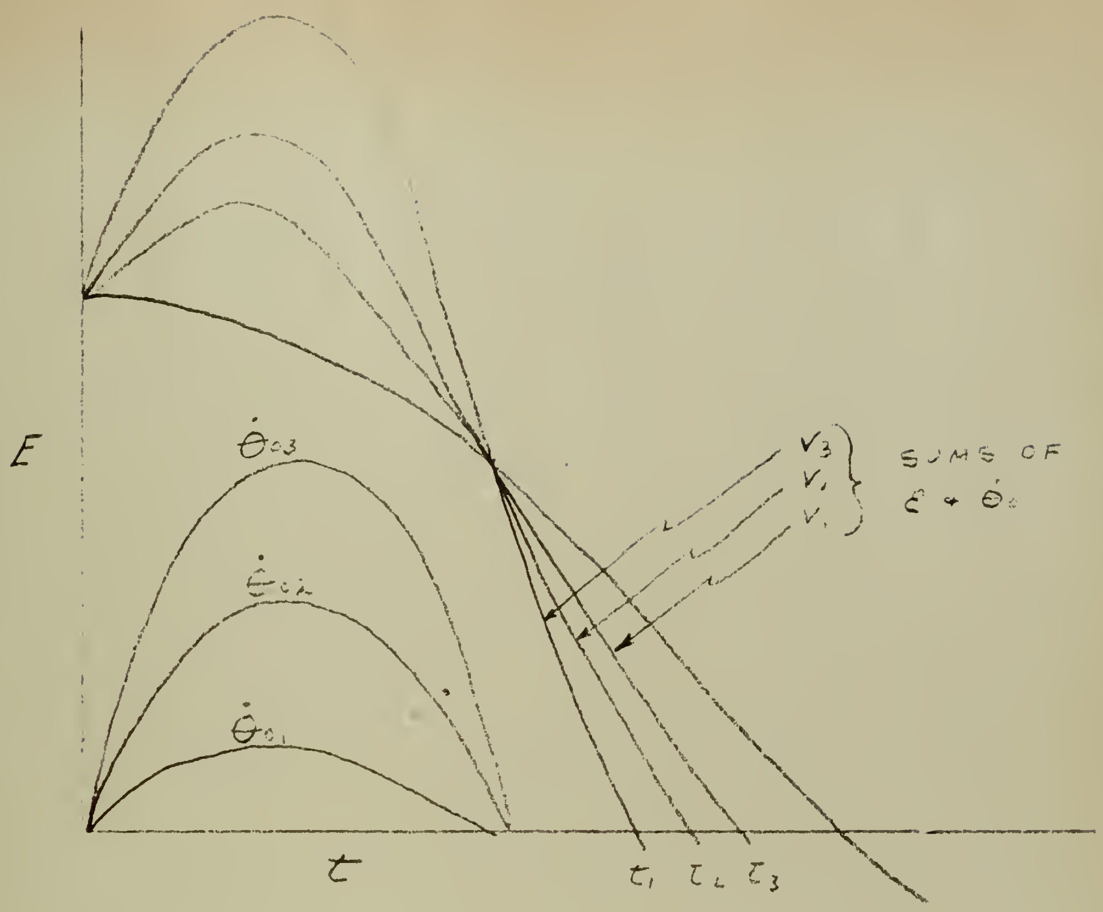


Fig. 5.2

The sum of the two voltages will pass through zero at some particular time. This time can be adjusted by varying the magnitude of $\dot{\theta}_o$ as shown in Fig. 5.2. The summed voltage (referred to as V) can be employed to control the operation of the tach channel or lead network switch. Fig. 5.3 shows a schematic for employing this method of control.

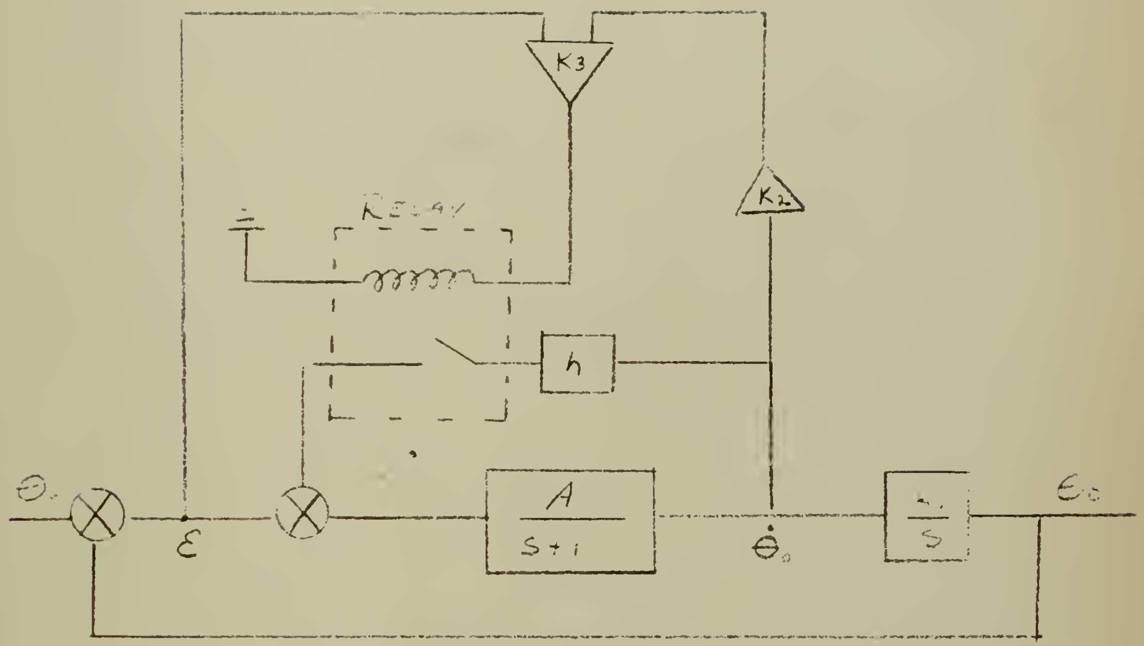


Fig. 5.3

K_2 in the above sketch is either an amplifier or an attenuator that is employed to control the amplitude of $\dot{\theta}_o$. K_3 is an adder amplifier that adds the two quantities $\dot{\theta}_o$ and ε . The relay is a polarized relay that will close when the voltage applied to it passes through zero from a positive value to a negative value. The actuated relay is actually the switch in the tach feedback channel.

A more detailed description of the relay is shown in Fig. 5.4.

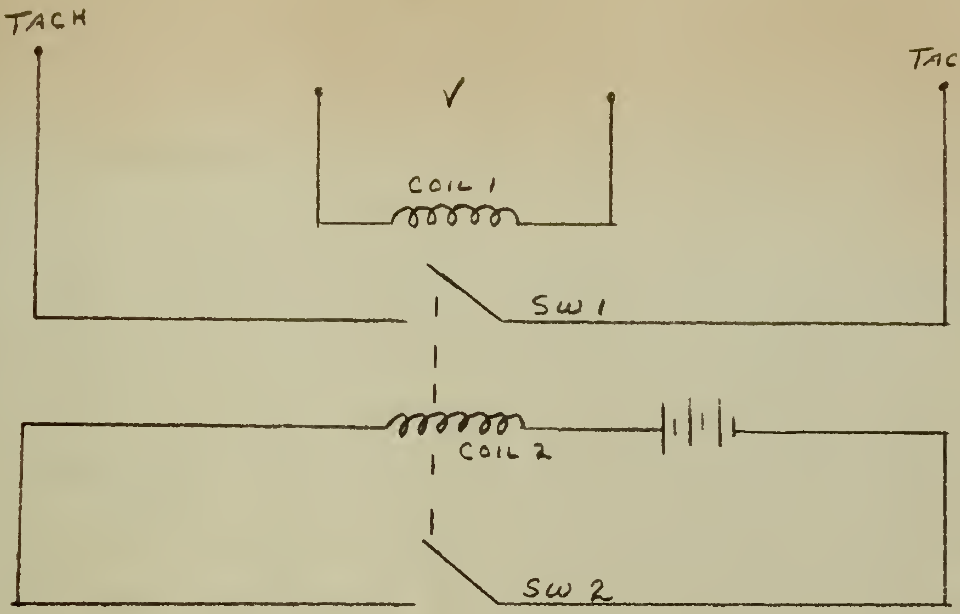


Fig. 5.4

Coil 1 is energized by the voltage V and closes the switch in the tach channel (sw 1) when V passes through zero. Another switch (sw 2) is mechanically ganged to sw 1 and closes at the same instant. When sw 2 closes it places a voltage across coil 2 which insures that sw 1 remains closed regardless of what happens to the voltage V.

Coil 2 and sw 2 are included here because once the tach channel switch closes the value of $\mathcal{E}$ and $\dot{\Theta}_o$ (and therefore V) will be altered in some way. The possibility exists that V will be altered in such a way that it will allow sw 1 to reopen. This cannot be tolerated so coil 2, sw 2, and the battery are included to prevent this from occuring.

B) MECHANICAL CONTROL

Chapter IV pointed out the disadvantage of an error in the control of the time delay. The accuracy of the automatic control is entirely dependent on the quality of the components used. Their accuracy is dependent on such variables as temperature humidity, age, etc. Therefore,

unless the components are quite expensive, a fluctuating error can be expected over a period of time.

A more accurate and dependable method of control was employed along with the automatic control in the preparation of this paper. It consists primarily of a series of contacts actuated by individual cams rotating on a common shaft. These cams are adjustable around the shaft allowing the cams to be in phase with each other operating the contacts in phase, or they can be rotated out of phase with each other, operating the contacts out of phase. The time between the closing of one contact and the closing of the next contact can then be accurately controlled by adjusting the speed of the shaft which is driven by a DC motor. This system will be referred to hereafter as the trigger mechanism or trigger contacts. A schematic of the trigger mechanism is shown in Fig. 5.5.

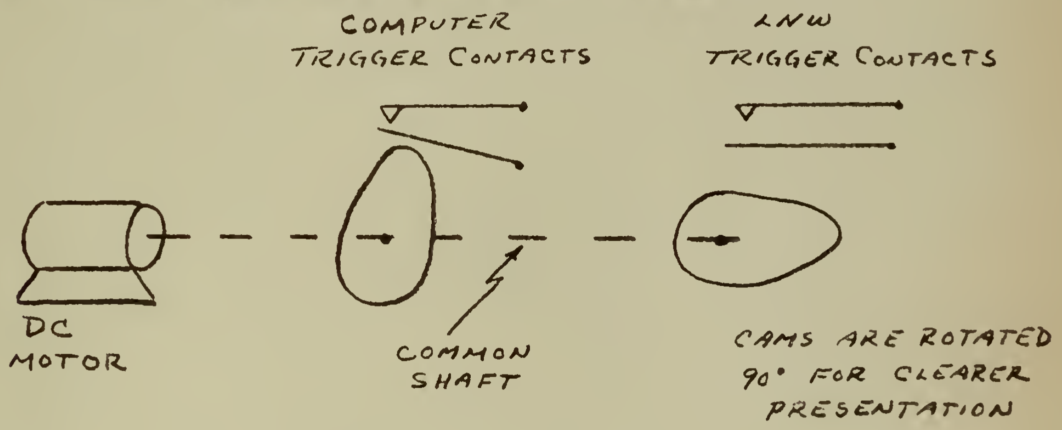


Fig. 5.5

The trigger system is then integrated with a relay system which operates the computer and closes the compensator channel switch at the desired time. The overall system for the lead network compensation is shown in Fig. 5.6. Minor variations will also adapt this to the tach feedback compensation.

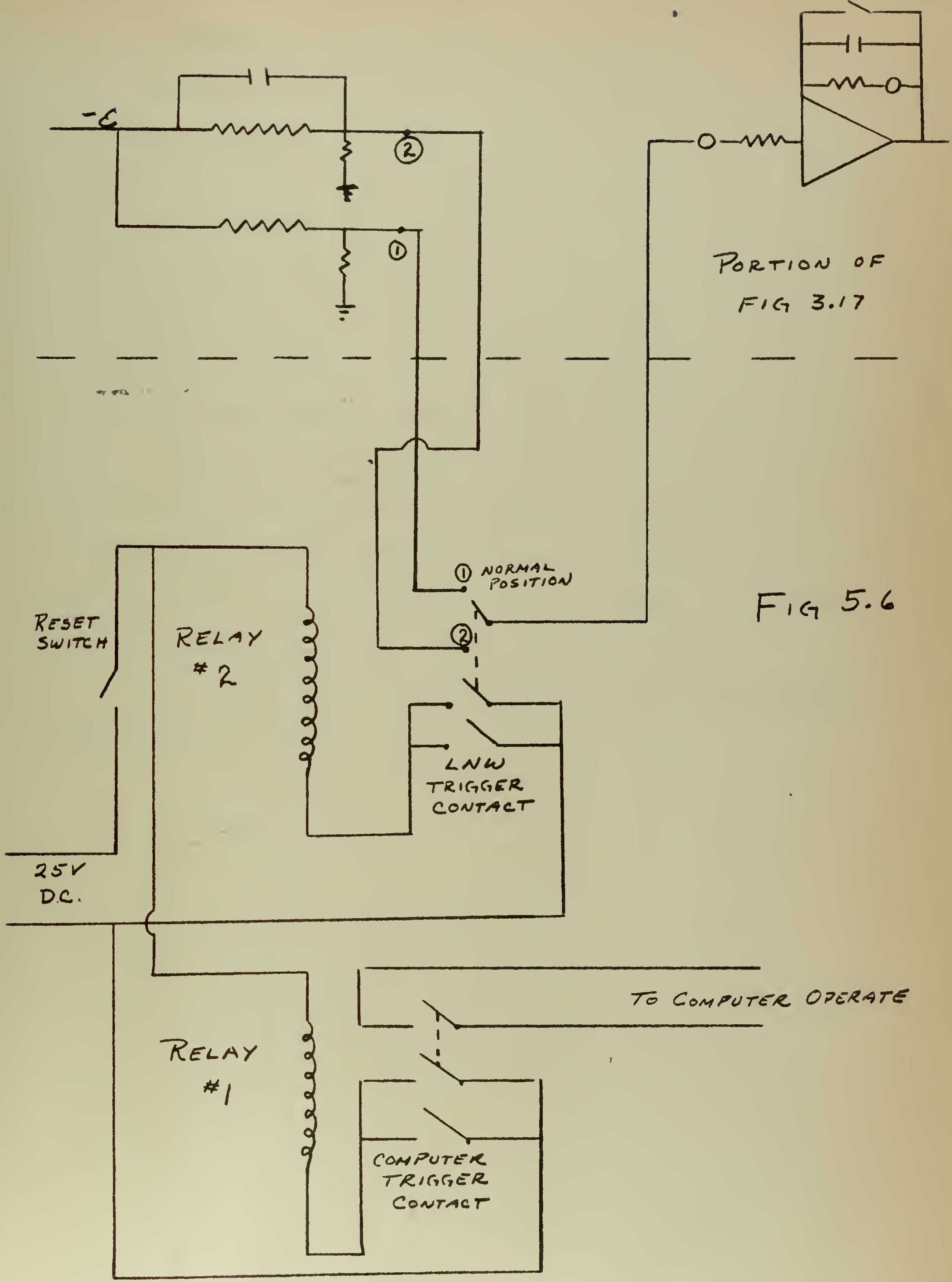

-C
②
①
PORTION OF
FIG 3.17
① NORMAL POSITION
②
RESET SWITCH
RELAY #2
LNW TRIGGER CONTACT
25V D.C.
TO COMPUTER OPERATE
RELAY #1
COMPUTER TRIGGER CONTACT

FIG 5.6

The problem is initiated by energizing the DC motor. The cams in Fig. 5.5 are positioned such that the computer trigger is the first contact t to close. Referring to Fig. 5.6 this energizes relay #1 closing the computer operate switch. The computer is now energized and is operating with a low ζ_1 because the parallel resistance circuit is connected to amplifier #3 and not the lead network. A switch in parallel with the computer trigger switch is also closed insuring that relay #1 remains energized after the cam allows the computer trigger contact to open.

After a time equal to T, the next cam closes the lead network (LNW) trigger contact energizing relay #2. This throws the two way switch from position 1 (normal position) to position 2 which places the lead network in the computer circuit. The LNW trigger contact also has another switch in parallel with it which insures that relay #2 remains closed after the cam allows the LNW trigger contact to open.

At the completion of the problem the reset switch will return the relay system to its original state.

VI SUGGESTIONS FOR FURTHER INVESTIGATION

A) A complete mathematical analysis of the transient condition occurring at the time of switching in the lead network systems.(see Chapter IIB 2).

B) A mathematical investigation of the optimum values of resistance in the parallel resistance network of the lead network systems. (see Chapter IIB 2)

C) Development of a better switching scheme for the lead network system.

D) Employment of a lag network in a delayed compensation system.

E) Experimental laboratory investigation of second and third order systems.

F) Phase plane analysis of second order tachometer feedback.

G) Phase plane analysis of second and third order lead network systems.

H) Further investigation of third order tachometer feedback systems with adjustable gain at the time of switching. (see Chapter IIIA 1)